Let \( A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \).

\( A^T A \) will be significant when we investigate the *singular value decomposition* (SVD). We proved that \( A^T A \) is symmetric. Symmetric matrices have real eigenvalues. \( A^T A \) is also positive semi-definite: it has no negative eigenvalues.

1. Find \( A^T A \).

2. Find \( (A^T A - \lambda I) \).

3. Find \( \det(A^T A - \lambda I) \).
   (\( \lambda \) is a variable—so your answer will have \( \lambda \)s in it).
4. Solve \( \det(A^T A - \lambda I) = 0 \).

5. For each solution \( \lambda \), write the equation \((A^T A - \lambda I)x = 0\), and solve for \( x \).

6. Check that your eigenvalue-eigenvector pairs work!