$A$ is an $m \times n$ matrix ($m$ rows, $n$ columns), with rank $r$. The **Four Subspaces** are $C(A)$, $C(A^T)$, $N(A)$ and $N(A^T)$.

**Fundamental Theorem of Linear Algebra, Part 1**

- The column space and row space both have dimension $r$.

- The nullspaces have dimension $n-r$ and $m-r$.

\[
A = \begin{bmatrix}
1 & 3 & 0 & 5 \\
0 & 0 & 1 & 6 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

1. What is the rank $r$ of $A$?

2. Find a *basis* for the row space of $A$.

3. What is the dimension of the row space of $A$?

4. Find a *basis* for the null space of $A$.

5. What is the dimension of the nullspace of $A$?

6. Check that the dimension of the nullspace of $A$ is $n-r$. 

7. Find $A^T$, then find the RREF for $A^T$, and a basis for the column space of $A$.

8. What is the dimension of the column space of $A$?

9. Find a basis for the null space of $A^T$.

10. What is the dimension of the nullspace of $A^T$?

11. Check that the dimension of the nullspace of $A^T$ is $n-r$.

**Fundamental Theorem of Linear Algebra, Part 2**

- The row space of $A$ and the null space of $A$ are orthogonal.

  (So the row space of $A^T$ and the null space of $A^T$ are orthogonal. So...)

- The column space of $A$ and the null space of $A^T$ are orthogonal.