Finding Bases.

For each of the row space, column space and null space of a matrix $A$ we want to find a linearly independent set of vectors whose linear combinations is that space. Such a set of vectors is called a basis for the space.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

1. Find the RREF for $A$.

2. Find the rank of $A$.

3. Show that the pivot rows are linearly independent. Thus they are a basis for the row space $C(A^T)$ of $A$.

4. The dimension of the row space is the number of vectors in a basis. What is the dimension of $C(A^T)$?
5. Use the RREF to find the null space \( N(A) \). The null space is a linear column of the special vectors. Show that the special vectors are linearly independent. What is the dimension of the null space?

You can find a basis for the column space of \( A \) by applying these ideas to \( A^T \).

6. Find the RREF for \( A^T \).

7. Find the rank of \( A^T \).

8. Show that the pivot rows are linearly independent. Thus they are a basis for the row space of \( A^T \)—which is also the column space \( C(A) \) of \( A \).

9. What is the dimension of \( C(A) \)?

10. Check that the dimension of the column space \( C(A) \) and the dimension of the null space \( N(A) \) sum to the number of columns of \( A \).