Null Space of a Matrix.

The *column space* $C(A)$ of a matrix $A$ is the set of all linear combinations of its columns.

Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

1. List the columns of $A$.

2. Write the definition for $C(A)$ in terms of these columns.

3. Can you find a vector that is not in the column space of $A$?

4. Can you find a “nice” description of $C(A)$?

The *row space* $C(A^T)$ of a matrix $A$ is the set of all linear combinations of its rows.

5. List the rows of $A$.

6. Write the definition for $C(A^T)$ in terms of these rows.
7. Can you find a vector that is not in the row space of $A$?

8. Can you find a “nice” description of $C(A^T)$?

9. Find a specific (non-trivial) vector $\vec{v}$ in the row space of $A$.

The null space $N(A)$ of a matrix $A$ is the set of all vectors $\vec{x}$ where $A\vec{x} = \vec{0}$.

10. Find $N(A)$ by solving $A\vec{x} = \vec{0}$.

11. Find a specific (non-trivial) vector $\vec{x}$ in the null space of $A$.

12. Check that $\vec{v} \cdot \vec{x} = 0$.

13. Can you find a “nice” description of $N(A)$?