The column space \( C(A) \) of a matrix \( A \) is the set of all linear combinations of its columns.

1. Describe the column space of \( A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)

2. Describe the column space of \( A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \). Can you find a vector that is not in the column space?

3. Describe the column space of \( A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix} \).
The row space $C(A^T)$ of a matrix $A$ is the set of all linear combinations of its rows.

4. Describe the row space of

$$A = \begin{bmatrix}
1 & 2 & 3 \\
0 & 0 & 4
\end{bmatrix}.$$

5. Find a specific (non-trivial) vector $\vec{v}$ in the row space of $A$.

The null space $N(A)$ of a matrix $A$ is the set of all vectors $\vec{x}$ where $A\vec{x} = \vec{0}$.

6. Describe the null space of

$$A = \begin{bmatrix}
1 & 2 & 3 \\
0 & 0 & 4
\end{bmatrix}.$$

7. Find a specific (non-trivial) vector $\vec{x}$ in the null space of $A$.

8. Check that $\vec{v} \cdot \vec{x} = 0$. 