LARSON—MATH 310—CLASSROOM WORKSHEET 18
Transpose—Column Space of a Matrix.

The *column space* of a matrix is the set of all linear combinations of its columns.

1. Let \( A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \). Find \( A^T \).

2. Find \( A^T A \).

3. Find \( AA^T \).

4. What do you notice about both \( A^T A \) and \( AA^T \)? Can you explain it?
5. Describe the column space of \( I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)

6. Describe the column space of \( I = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \). Can you find a vector that is not in the column space?

7. Describe the column space of \( I = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix} \).