LU-Decomposition.

Fact: The product of lower-triangular matrices is lower-triangular.

1. Let $L_1 = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 4 & 0 \\ 5 & 6 \end{bmatrix}$. Find $L_1 L_2$ and check that it is lower-triangular.

2. Let $L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ and $L_2 = \begin{bmatrix} -1 & 0 & 0 \\ 5 & 6 & 0 \\ 1 & 2 & 3 \end{bmatrix}$. Find $L_1 L_2$ and check that it is lower-triangular.

Fact: The inverse of an (invertible) lower-triangular matrices is lower-triangular.

3. Find the inverse of the lower-triangular matrix $L_1 = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$.

An LU-factorization of a matrix $A$ is a lower-triangular matrix $L$ and an upper-triangular matrix $U$ so that

$$A = LU.$$ 

The above facts—and elimination matrices—are what we need to show that any matrix that can be reduced to upper-triangular form without row switches admits an LU-factorization.