Last name ______________________

First name ______________________

LARSON—MATH 305—SAGE WORKSHEET 04
Primes.

1. Log in to your Sage Cloud account.
   (a) Start Firefox or Chrome browser.
   (b) Go to `http://cloud.sagemath.com`
   (c) Click "Sign In".
   (d) Click project **Math 305**.
   (e) Click "New", call it **s04**, then click "Sage Worksheet".

2. Evaluate `prime_range(10,50)` to find the primes from 10 up to and *not* including 50.

3. Find the primes from 1000 to 1050.

4. Evaluate `prime_pi(100)` to find the number of primes up to and including 100. This is Sage's implementation of $\pi(x)$.

5. Determine the number of primes up to 1,000 and then up to 1,000,000.

6. To plot $\pi(x)$ for x from 50 to 100, evaluate: `plot(prime_pi, 50, 100)`.

7. Sketch $\pi(x)$ for x from 100 to 1000.

8. To check the primality of 53, evaluate: `is_prime(53)`.

9. Determine if 1234567 is prime. How about 123456789?

10. You can define a function that gives the *Fermat numbers* $F\_n = 2^{2^n} + 1$ with `F(n)=2**(2**n)+1`. Find $F(0)$, $F(1)$, $F(2)$, $F(3)$, $F(4)$, and $F(5)$.

11. Check which of these are prime.

12. To find the factors of an integer n, evaluate: `factor(n)`. Find the factors of 1001001.

13. If any of the Fermat numbers you calculated were composite, find the factors.

14. The Prime Number Theorem says that $\lim_{x\to\infty} \frac{\pi(x)}{\frac{x}{\log x}} = 1$. We can investigate this limit numerically. Evaluate `PNT(n)=prime_pi(n)*log(n)/n`. Then evaluate PNT(1,000), PNT(1,000,000), and PNT(1,000,000,000).

15. You can get interesting graphs of this function too. Evaluate: `plot(PNT,10,1000)` and `plot(PNT,10,1000000)`. .

16. Let $p_1 = 2, p_2 = 3$ be the first two *Euclidean primes*. The k^{th} Euclidean prime is defined to be the smallest prime factor of $P_k = (p_1 \cdot p_2 \cdot \ldots \cdot p_{k-1}) + 1$. So $P_3 = 2 \cdot 3 + 1$, and $p_k = 7$. Find P_4 and p_4.

17. Find the next few Euclidean primes: p_5, p_6 and p_7. Use the `factor` command as needed.