Suppose \( a = nq + r \)

\[ \text{claim: } a \equiv r \pmod{n} \]

That is, \( n \mid (a-r) \) by the definition of congruence.

So, \( n \mid [(nq+r)-r] \)

\[ \implies n \mid nq \]

which is true.

So, reverse these steps to construct a proof.

Suppose \( a \equiv b \pmod{n} \)

and \( b \equiv c \pmod{n} \)

That is, \( n \mid (a-b) \), by def., and \( n \mid (b-c) \).

So, \( \exists k_1 \) such that \( n k_1 = a-b \)

and \( \exists k_2 \in \mathbb{Z} \) such that \( n k_2 = b-c \).
Then \( nK_1 + nK_2 = (a-b) + (b-c) \)
and \( n(K_1 + K_2) = a-c \)
so \( n | (a-c) \).

and \( a \equiv c \pmod{n} \),
which was to be shown.

5, 6, 7 Done in class

8 Done in Sage — by
making a multiplication table —
every row contains a “1.”