Math 305 - Homework #2

(1) Suppose
$a = nq + r$

<u>claim</u>: $a \equiv r \pmod{n}$

That is, $n \mid (a-r)$ by the definition of congruence

So, $n \mid [(nq+r)-r]$

$\Leftrightarrow n \mid nq$

which is true.

So, <u>reverse</u> these steps to construct a proof.

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(4) Suppose $a \equiv b \pmod{n}$
and $b \equiv c \pmod{n}$

That is, $n \mid (a-b)$, by def.,
and $n \mid (b-c)$

So, $\exists k_1$ such that $nk_1 = a-b$
and $\exists k_2 \in \mathbb{Z}$ such that $nk_2 = b-c$

Then $nk_1 + nk_2 = (a-b) + (b-c)$

and $n(k_1 + k_2) = a - c$

so $n \mid (a-c)$

and $a \equiv c \pmod{n}$,

which was to be shown. ✓

5, 6, 7 Done in class

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8 Done in Sage - by making a multiplication table - every row contains a "1."