Organizational Notes

1. A Zoom recording link and class notes will be sent out after each Zoom class.

2. Remember to send your answers to the classroom worksheets. Title your email with enough to help me record your “participation”.

3. Homework #6 is due Mon., Nov. 16.

Review

1. A non-zero $d$ is a quadratic residue (or square) in $\mathbb{Z}/p\mathbb{Z}$ if there is an $x \in \mathbb{Z}/p\mathbb{Z}$ such that $x^2 \equiv d \pmod{p}$.

2. The Legendre symbol $\left( \frac{a}{p} \right)$ is defined to be 0 if $\gcd(a,p) > 1$, 1 if $a$ is a quadratic residue in $\mathbb{Z}/p\mathbb{Z}$, and $-1$ otherwise.

Quadratic Reciprocity (Chapter 4)

1. (Multiplicity of Legendre’s Symbol). Show: $\left( \frac{ab}{p} \right) = \left( \frac{a}{p} \right) \left( \frac{b}{p} \right)$ (where $p$ is prime).

2. What implications does the proof of this claim have?

3. (Gauss’ Quadratic Reciprocity Law) For distinct odd primes $p$ and $q$,

$$\left( \frac{q}{p} \right) = (-1)^{\frac{p-1}{2} \cdot \frac{q-1}{2}} \left( \frac{p}{q} \right)$$

Test this statement for small values of $p$ and $q$. 
4. Use Gauss’ Quadratic Reciprocity Law to compute \( \left( \frac{15}{53} \right) \).

5. Use Gauss’ Quadratic Reciprocity Law to compute \( \left( \frac{3}{12345678910987654321} \right) \)

   (12345678910987654321 is indeed prime).

6. (Euler’s Criteria). \( a^{p-1} \equiv \left( \frac{a}{p} \right) \) (mod p), for prime p. Check for \( p = 13 \).

**Major Theory Plan:**

1. Existence of units.
2. Chinese Remainder Theorem.
3. \( \phi \) is multiplicative.
5. Quadratic Residues & Legendre Symbol (Chp.4)
6. Euler’s Criterion.
7. Gauss’ Lemma
8. Quadratic Reciprocity.