Last name _____

First name _____

LARSON—MATH 305—CLASSROOM WORKSHEET 10

Rings: $\mathbb{Z}, n\mathbb{Z}, \mathbb{Z}/n\mathbb{Z},$

Organizational Notes

1. A Zoom recording link and class notes will be sent out after each Zoom class.

2. Remember to send your answers to the classroom worksheets. Title your email with enough to help me record your "participation".

Review

1. (**Fundamental Theorem of Arithmetic**) Show that every natural number is a *unique* product of primes.

2. What is a *ring*?

3. Show that $\mathbb{Z}[\sqrt{-5}] = a + b\sqrt{-5} : a, b \in \mathbb{Z}$ does *not* have unique factorization.

Chapter 2—Rings

1. Why is $\mathbb{Z}$ a ring?

2. What is $n\mathbb{Z}$?

3. Explain why $n\mathbb{Z}$ is a ring.

4. Suppose $a \equiv b \pmod{n}$. Let $a = nq + r$ ($0 \leq r \leq |b|$, from the Division Algorithm). Explain why $a \equiv r \pmod{n}$.

5. What is $\mathbb{Z}/n\mathbb{Z}$?

6. Explain why $\mathbb{Z}/n\mathbb{Z}$ is a ring.

7. What is a *field*?

8. What is an example of a field?