1. Log in to your Sage/Cocalc account.
   
   (a) Start the Chrome browser.
   (b) Go to http://cocalc.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it c38, then click “Sage Worksheet”.

Here are the Final-type questions, together with a selection of problems. Go over your classroom worksheets as needed. You will be allowed to refer to your classroom worksheets on the Final.

2. Write a program to find the set of positive integers less than 100 that are multiples of 3 or 5.

3. Write a Sage Interact that takes any function as input (so use an input box) and plots the graph of that function. Choose any function as the default input.

4. Write a Sage Interact that takes $c$ as input and plots the graph of $f(x) = \sin(x)$ on the interval $[0, c]$. Use a slider to allow the user to choose any $c$ from 1 to 100. Let $c=5$ be the default.

5. Write a Sage Interact that plots $f(x) = \sin(x)$ on $[0, 2\pi]$ and uses a dropdown menu that allows the user to choose between the colors red, blue and green.

6. Define a function \texttt{random\_average}(n) to choose $n$ random integers between 1 and 100 and find the average of these $n$ numbers. Find \texttt{random\_average}(n) for $n = 10$ to $n = 1000$. Use \texttt{scatter\_plot} to display the results.
7. Define a function \texttt{order\_size}(g) which takes a graph \textit{g} as input and returns the product of the number of vertices of the graph and the number of edges of the graph.

8. The prime factors of 13195 are 5, 7, 13 and 29. Write a function \texttt{largest\_prime\_factor}(n) which returns the largest prime factor of a given integer \textit{n}. What is the largest prime factor of the number 600851475143?

9. If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Write a program to find the sum of all the multiples of 3 or 5 below 1000.

10. When \( n = 0 \), \( n^2 - 79n + 1601 \) is 1601—which is prime. When \( n = 1 \), \( n^2 - 79n + 1601 \) is 1523—which is prime. Find the smallest value of \( n \) where \( n^2 - 79n + 1601 \) is not prime.

11. 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

12. The Fibonacci sequence \( F_n \) is defined as follows \( F_0 = 0, F_1 = 1 \) and \( F_n = F_{n-1} + F_{n-2} \) for \( n > 1 \). What is the first term in the Fibonacci sequence to contain 1000 digits?

13. (Ramanujan) 2, 9, 16, etc. can be written (uniquely) as the sum of 2 cubes \((1^3 + 1^3, 1^3 + 2^3, 2^3 + 2^3, \text{etc.})\). Find the smallest integer which can be written as the sum of 2 cubes in 2 different ways.