LARSON—MATH 255–CLASSROOM WORKSHEET 35
Problems & Interacts

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c35**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Problems

2. A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which, $a^2 + b^2 = c^2$, For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. There exists exactly one Pythagorean triplet for which $a + b + c = 1000$. **Find** the product abc.

3. The sum of the squares of the first ten natural numbers is, $1^2 + 2^2 + \ldots + 10^2 = 385$.

   The square of the sum of the first ten natural numbers is, $(1 + 2 + \ldots + 10)^2 = 55^2 = 3025$.

   Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$.

   Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

More Interacts!

There is a collection of examples of Sage Interacts at `http://wiki.sagemath.org/interact/`. Let's look at a few of these examples to see the kinds of things you can do with Sage.

Posets

4. A *poset* is a very common mathematical object. They consist of a set together with a *relation* that is reflexive, transitive, and anti-symmetric. Any collection of lists or sets with the subset relation form a poset. Try `P=Poset([[1,2],[],[1]])`. This makes a Poset P consisting of 3 lists. You can get a nice picture (called a *Hasse diagram*) of this poset with the command `P.show()`.

5. Consider the list of integers `L=[5..10]`. Ordinary inequality defines a relation on L. So (a, b) is in the relation if and only if $a \leq b$. Evaluate:
`Q=Poset(([5..10], lambda x, y:  x<=y))`. Then `show` it.

6. Can you think of another relation on the positive integers? How about "$\geq$". Experiment that—and make a picture.

7. The positive integers together with the relation R where a pair (a, b) is in R if and only if a divides b is a relation. So, for instance, $(1, 5)$ is in R as 1 divides 5 and $(2, 4)$ is in R as 2 divides 4. Here's a Sage INTERACT that makes a nice picture (called a *Hasse diagram*) of the positive integers with the divisibility relation.

```
@interact
def _(n=(5..100)):
    Poset(([1..n], lambda x, y: y%x == 0) ).show()
```

8. Define any other Poset in Sage and make a Hasse diagram for that Poset. (Here's one you could try: the subsets of a set form a poset. Could you code that?)

Random Walks

9. Start at the origin on the number line. At each time step take a (random) step one unit to the right or one unit to the left. I have heard that you will (with probability 1) return to the origin at some point, Is this true? How can we investigate this experimentally?

10. If it is true, how many steps does it take on average to return to the origin?

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255—c35 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!