LARSON—MATH 255–CLASSROOM WORKSHEET 34
The Birthday Problem

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c34**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

The Birthday Problem.

2. (Guess) **How many students do we need in a classroom so that there is a better than 50% chance that at least one pair of them have the same birthday (Month & Day)?**

3. What could you code to investigate this problem?

   What We've Done So Far

   There were 17 students in class. What is the probability that some pair of them will have the same birthday?

   We made 2 simplifying assumptions: (1) All years have 365 days, and (2) that all birthdays are equally likely.

   (a) We let the numbers from 0 to 354 represent the days of the year.
   (b) We chose 17 random "birthdays".
   (c) We figured out how to check if at least two of the numbers are the same.

   This is one *experiment*.

   What We'd Still Like to Do

   (a) Repeat this experiment lots of times.
   (b) Record the number of times that there were at least one pair of same birthdays.
   (c) Then divide that number by the number of experiments to get an (empirical) probability.

   Then We Can Generalize

   All that work was for finding the probability that some pair of 17 students have the same birthday. But there was nothing special about that number of students—we can repeat all this for any number n of students. **So we should write functions to both conduct one experiment with n students, and to repeat an experiment with m students (so we can vary the number of experiments)**.

4. Now make a `scatter_plot` to visualize the probability as the number of students goes from 1 to 365.

Problems

5. A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which, $a^2 + b^2 = c^2$, For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. There exists exactly one Pythagorean triplet for which $a + b + c = 1000$. **Find** the product abc.

6. The sum of the squares of the first ten natural numbers is, $1^2 + 2^2 + \ldots + 10^2 = 385$.

   The square of the sum of the first ten natural numbers is, $(1 + 2 + \ldots + 10)^2 = 55^2 = 3025$.

   Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$.

   Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

7. The number, 197, is called a *circular prime* because all rotations of the digits: 197, 971, and 719, are themselves prime. There are thirteen such primes below 100: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97. **How many** circular primes are there below one million?

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255—c34 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!