LARSON—MATH 255–CLASSROOM WORKSHEET 30
Stable Sets & Stability Number

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c30**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

2. **Ramanujan!** 2, 9, 16, etc. can be written (uniquely) as the sum of 2 cubes (1^3+1^3, 1^3+2^3, 2^3+2^3, etc.). Find the smallest integer which can be written as the sum of 2 cubes in 2 different ways.

   Can we do better than our original not-super-fast algorithm?

Stable Sets & Stability Number

Now we will define a new graph function for computing the hard-to-compute *stability number* of a graph. The *stability number* of a graph is the largest number of vertices in the graph that have no edges between them.

3. Find the stability number of `c5=graphs.CycleGraph(5)` by hand. What is the largest stable set of vertices that you can find?
4. Find the stability number for `p2=graphs.PathGraph(2)`. Use `p2.show()` to help you visualize.
5. Find the stability number for `p3=graphs.PathGraph(3)`. Use the `show()` method to help you visualize.
6. Find the stability number for `p4=graphs.PathGraph(4)`. Use the `show()` method to help you visualize.
7. Find the stability number for `p5=graphs.PathGraph(5)`. Use the `show()` method to help you visualize.
8. Find the stability number for `c5=graphs.CompleteGraph(5)`. Use `c5.show()` to help you visualize.
9. Find the stability number for `g=graphs.PetersenGraph()`.
10. *Prove* that the stability number for the Petersen graph is 4. To do this you need two things: (1) a stable set with 4 vertices, and (2) an argument that no stable set can have more than 4 vertices.

Now we will write an algorithm to find the stability number.

11. First let's write a test for whether or not a specific set S of vertices is stable. Then one possible algorithm involves testing every subset of vertices. If S is stable then the test for (i, j) will not be an edge (will not be in the list of edges) of graph g for each possible pair i, j of vertices.

```
def is_stable(g, S):
    E=g.edges(labels=False)
    for i in S:
        for j in S:
            if (i,j) in E:
                return False
    return True
```

12. Test this for some (sub)sets of vertices of the Petersen graph.

Subsets and Counting

The "naive" (and inefficient) way to find a largest stable set in a graph is to test every subset of vertices, check if it is stable, and then keep track of the largest one you've seen up to that point. Let's see why its inefficient.

13. Suppose you have a very small graph, with just 3 vertices. How many subsets of the vertex set would you have to look at? Run: `Subsets([0,1,2])`.

14. How many is that? We can turn the *Subset generator* into a list and use the Python list counting function *len*. Run *len(Subsets([0,1,2]))*. (Don't ever try this on a set that's much larger—the list gets created in memory and might freeze your computer as it runs out of RAM).

15. How many subsets will we create for a graph that has n vertices? For each vertex v we will generate every possible set containing v and every possible set not containing v. So that will be 2 possibilities for each of the n vertices. That's 2^n subsets. (For our earlier example, thats $2^3 = 8$ subsets).

16. Here is a first function to find a maximum stable set of a graph.

```
def naive_maximum_stable_set(g):
    stable = []
    L=subsets(g.vertices())
    for S in L:
        if is_stable(g,S)==True:
            if len(S) > len(stable):
                stable = S
    return stable
```

17. Now use this function to find a maximum stable set of every graph you previously hand-calculated. Can you *prove* the set is maximum?

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255—c30 worksheet attached" (so that it will be properly recorded).