

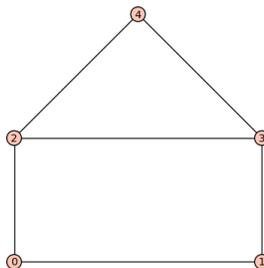
LARSON—MATH 255—CLASSROOM WORKSHEET 27
Graphs & Problems

1. (a) Start the Chrome browser.
(b) Go to `http://cocalc.com`
(c) Login using **your VCU email address** .
(d) Click on our class Project.
(e) Click “New”, then “Worksheets”, then call it **c27**.
(f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Graphs & Graph Theory

A **graph** is a mathematical object consisting of *dots* and *lines* (also called *vertices* and *edges*). The *order* of a graph is the number of vertices it has. The *size* of a graph is the number of edges it has. We can create our own graph using the `Graph()` constructor, and the `add_vertex()` and `add_edge()` methods.

2. Make the following graph, called “the house”. Start by letting `house=Graph(5)`. When you are done you can view it with `house.show()`.



Another way to represent a graph with order n is with an $n \times n$ *adjacency matrix* A . If the vertices of the graph are $\{v_0, v_1, \dots, v_{n-1}\}$ (or $\{1, 2, \dots, n-1\}$ for short) then the $A_{i,j}$ is 1 if there is an edge from vertex i to vertex j , and 0 if there is not.

3. Try:

```
g=graphs.PetersenGraph()
g.show()
g.adjacency_matrix()
```

Make sure you understand the pattern of 0's and 1's.

4. Try:

```
house.show()
house.adjacency_matrix()
```

Make sure you understand the pattern of 0's and 1's.

5. One way to make a graph is to start with a number of vertices and then for each pair of vertices n and m , flip a coin to decide whether to put an edge between those vertices. Try this:

```
g=Graph(10)
for i in [0..9]:
    for j in [0..9]:
        if i<j and random() < 0.5:
            g.add_edge(i,j)
g.size()
g.show()
```

6. The study of *random graphs* is huge and important and was initiated in a 1959 paper of Erdős and Renyi. Sage has a built in function to do this: `graphs.RandomGNP(n,p)`, where n is the number of vertices you want, and p is the probability of an edge ($0 \leq p \leq 1$). To simulate a coin flip, use $p = .5$. Run the following code a few times.

```
g=graphs.RandomGNP(10, 0.5)
g.size()
g.show()
```

Problems

7. **Goldbach!** Goldbach also conjectured that every odd composite number can be written as the sum of a prime and twice a square. So, $9 = 7 + 2 * 1^2$, $15 = 7 + 2 * 2^2$, $21 = 3 + 2 * 3^2$, etc.

It turns out that the conjecture was false. What is the smallest odd composite that *cannot* be written as the sum of a prime and twice a square?

8. **Ramanujan!** 2, 9, 16, etc. can be written (uniquely) as the sum of 2 cubes ($1^3 + 1^3$, $1^3 + 2^3$, $2^3 + 2^3$, etc.). Find the smallest integer which can be written as the sum of 2 cubes in 2 different ways.

9. **Challenge.** Try the problem at <http://projecteuler.net/problem=8>. How will you get that number into your program??? Do not type it in by hand.

Getting your classwork recorded

When you are done, before you leave class...

- Click the “Make pdf” (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then “Open”, then print or make a pdf using your browser).
- Send me an email with an informative header like “Math 255—c27 worksheet attached” (so that it will be properly recorded).
- Remember to attach today’s classroom worksheet!