LARSON—MATH 255–CLASSROOM WORKSHEET 26
Graphs & Problems

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c26**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Graphs & Graph Theory

A **graph** is a mathematical object consisting of *dots* and *lines* (also called *vertices* and *edges*). A **tree** is a graph that contains no cycles.

Sage includes the `graphs` class which contains a number of *methods*. Some of these include constructors for making well-known graphs.

2. Run:

```
g=graphs.PetersenGraph()
g.show()
```

3. The *order* of a graph is the number of vertices it has. The *size* of a graph is the number of edges it has. How many vertices and edges does the Petersen graph have? Run `g.order()` and `g.size()`.

4. We can create our own graph using the `Graph()` constructor, and the `add_vertex()` and `add_edge()` methods. Lets make a `cycle` on 5 vertices. First initialize the graph and make the vertices:

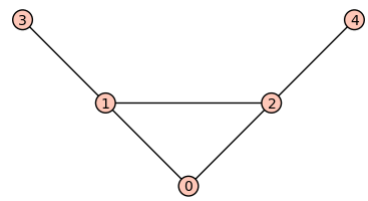
```
g=Graph()
for i in [1..5]:
    g.add_vertex()
g.show()
```

Notice that the vertex labels start at 0. Now make the edges:

```
for i in [0..3]:
    g.add_edge(i,i+1)
g.show()
```

You're still missing an edge. So add that.

5. Now use `Graph()`, `add_vertex()` and `add_edge()` to make the *bull*:



Start by letting `bull=Graph(5)`. Instead of using `add_vertex()`, you can start with `Graph(5)` to get a graph with 5 vertices and no edges. Now add the edges that you see in the diagram of the bull using `bull.add_edge()`. Remember that the layout of the graph doesn't matter—only that it has the same edges.

Problems

6. **Goldbach!** Goldbach conjeectured that ever even number larger than 2 is the sum of two primes. So $= 2 + 2$, $6 = 3 + 3$, $8 = 5 + 3$, etc. Despite much work (with real progress in the last 100 years) the conjecture remains unresolved (open). It is known to be true up to some ginormous n.

   Write a program `goldbach(n)` that takes an even integer greater than 2 as input and returns two primes p_1 and p_2 so that $n = p_1 + p_2$.

7. It was also proposed by Goldbach that every odd composite number can be written as the sum of a prime and twice a square. So, $9 = 7 + 2 * 1^2$, $15 = 7 + 2 * 2^2$, $21 = 3 + 2 * 3^2$, etc.

   It turns out that the conjecture was false. What is the smallest odd composite that *cannot* be written as the sum of a prime and twice a square?

8. **Ramanujan!** 2, 9, 16, etc. can be written (uniquely) as the sum of 2 cubes ($1^3 + 1^3$, $1^3 + 2^3$, $2^3 + 2^3$, etc.). Find the smallest integer which can be written as the sum of 2 cubes in 2 different ways.

9. **Challenge**. Try the problem at http://projecteuler.net/problem=8. How will you get that number into your program??? Do not type it in by hand.

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255—c26 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!