LARSON—MATH 255—CLASSROOM WORKSHEET 26
Probabilistic Primality Testing

1. Log in to your Sage/Cocalc account.
   
   (a) Start the Chrome browser.
   (b) Go to http://cocalc.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it c26, then click “Sage Worksheet”.

Problems

2. Find the sum of the even Fibonacci numbers less than four million.

3. By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13. What is the 10,001st prime number?

4. $n!$ means $n \times (n-1) \times \ldots \times 3 \times 2 \times 1$. For example, $10! = 10 \times 9 \times \ldots \times 3 \times 2 \times 1 = 3628800$, and the sum of the digits in the number 10! is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$. Find the sum of the digits in the number 100!
5. The most obvious way to check if an integer \( n \) is prime is to divide it by every integer \( a \) with \( 1 < a < n \). If \( a \) divides \( n \) evenly (that is, has remainder 0) then \( n \) is not prime.

```python
def primality_test1(n):
    for a in [2..(n-1)]:
        if n%a==0:
            print "{} is not prime".format(n)
            return
    print "{} is prime".format(n)
```

Test this with some values of \( n \) where you know what the answer should be.

6. Notice that, if \( n \) is divisible by \( a \) (so \( \frac{n}{a} \) is an integer), then one of \( a \) or \( \frac{n}{a} \) must be no more than \( \sqrt{n} \) (if they were both bigger, their product would be bigger than \( n \)). This means we can improve our last test by only checking whether integers \( a \leq \sqrt{n} \) divide \( n \) evenly.

```python
def primality_test2(n):
    largest = floor(sqrt(n))
    for a in [2..largest]:
        if n%a==0:
            print "{} is not prime".format(n)
            return
    print "{} is prime".format(n)
```

Test this with some values of \( n \) where you know what the answer should be.

7. Now let’s compare the speeds of these two programs. 1777771 is a 7-digit prime number. Try `timeit("primality_test1(1777771)"` and `timeit("primality_test2(1777771)"")`