LARSON—MATH 255–CLASSROOM WORKSHEET 24
Problems

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click "New", then "Worksheets", then call it **c24**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Problems

2. By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13. What is the 10,001st prime number?

3. Let $p_1, p_2, \ldots, p_k$ be a list of any k primes. The product p of these primes plus one is

$$p = p_1 \cdot p_2 \cdot \ldots p_k + 1$$

   p is either a prime (different from each of these k primes) or it has a prime factor also different from each of these. (This implies there are infinitely many primes). Write a program to find the smallest prime number q no more than p and different from each of $p_1, p_2, \ldots, p_k$.

4. The Fibonacci sequence F_n is defined as follows $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$. What is the first term in the Fibonacci sequence to contain 1000 digits?

5. Find the sum of the even Fibonacci numbers less than four million.

6. $n!$ means $n \times (n-1) \times \ldots \times 3 \times 2 \times 1$. For example, $10! = 10 \times 9 \times \ldots \times 3 \times 2 \times 1 = 3628800$, and the sum of the digits in the number 10! is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$. Find the sum of the digits in the number 100! (100-factorial).

Goldbach!

7. (Goldach's Conjecture). Goldbach conjeectured that ever even number larger than 2 is the sum of two primes. So $= 2 + 2$, $6 = 3 + 3$, $8 = 5 + 3$, etc. Despite much work (with real progress in the last 100 years) the conjecture remains unresolved (open). It is known to be true up to some ginormous n.

   Write a program `goldbach(n)` that takes an even integer greater than 2 as input and returns two primes p_1 and p_2 so that $n = p_1 + p_2$.

8. It was also proposed by Goldbach that every odd composite number can be written as the sum of a prime and twice a square. So, $9 = 7 + 2 * 1^2$, $15 = 7 + 2 * 2^2$, $21 = 3 + 2 * 3^2$, etc.

   It turns out that the conjecture was false. What is the smallest odd composite that *cannot* be written as the sum of a prime and twice a square?

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255—c23 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!