

LARSON—MATH 255—CLASSROOM WORKSHEET 23

Problems

1. (a) Start the Chrome browser.
(b) Go to <http://cocalc.com>
(c) Login using **your VCU email address**.
(d) Click on our class Project.
(e) Click “New”, then “Worksheets”, then call it **c23**.
(f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be **#Problem 2**.

Problems

2. The sum of the reciprocals of the positive integers

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges (that is, the sum goes to infinity).

- (a) Find the smallest integer m so that $\sum_{n=1}^m \frac{1}{n}$ is at least 2.
(b) Find the smallest integer m so that $\sum_{n=1}^m \frac{1}{n}$ is at least 3.
(c) Find the smallest integer m so that $\sum_{n=1}^m \frac{1}{n}$ is at least 4.
3. By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13. What is the 10,001st prime number?
4. Let p_1, p_2, \dots, p_k be a list of any k primes. The product p of these primes plus one is

$$p = p_1 \cdot p_2 \cdot \dots \cdot p_k + 1$$

p is either a prime (different from each of these k primes) or it has a prime factor also different from each of these. (This implies there are infinitely many primes). Write a program to find the smallest prime number q no more than p and different from each of p_1, p_2, \dots, p_k .

5. The Fibonacci sequence F_n is defined as follows $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$. What is the first term in the Fibonacci sequence to contain 1000 digits?
6. Find the sum of the even Fibonacci numbers less than four million.
7. $n!$ means $n \times (n-1) \times \dots \times 3 \times 2 \times 1$. For example, $10! = 10 \times 9 \times \dots \times 3 \times 2 \times 1 = 3628800$, and the sum of the digits in the number $10!$ is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$. Find the sum of the digits in the number $100!$ (100-factorial).

Getting your classwork recorded

When you are done, before you leave class...

- (a) Click the “Make pdf” (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then “Open”, then print or make a pdf using your browser).
- (b) Send me an email with an informative header like “Math 255—c23 worksheet attached” (so that it will be properly recorded).
- (c) Remember to attach today’s classroom worksheet!