1. Log in to your Sage/Cocalc account.
   
   (a) Start the Chrome browser.
   (b) Go to http://cocalc.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it **c21**, then click “Sage Worksheet”.

**More Interacts**

2. `is_prime(n)` is a built-in Sage function which tests if an integer $n$ is prime. We can use it to write a function for counting the number of primes which are no more than $n$.

   ```python
   def count_primes(n):
       count = 0
       for x in [2..n]:
           if is_prime(x) == True:
               count = count + 1
       return count
   ```

   Find `count_primes(100)`, `count_primes(10000)`, and `count_primes(1000000)`.

3. `prime_pi(n)` is a built-in Sage function which does the same thing as `count_primes(n)`. Compute `prime_pi(100)`, `prime_pi(1000)`, and `prime_pi(1000000)`. Check that you get the same results as before.

4. Now let's compare the speeds of our prime counting function and the built-in prime counting function. Try `timeit("prime_pi(100000)")` and `timeit("count_primes(100000)")`. Which is faster?
5. Try the following Sage Interact which visualizes the Prime Number Theorem (PNT).

```python
@interact
def pnt(N=input_box(200)):
    show(plot(prime_pi,0,N,color='red')+plot(x/(log(x)-1),5,N,color='blue'))
```

Try putting different numbers in the input box.

6. Let’s do the same thing in a new way. Try:

```python
@interact
def pnt2(N=(100,(2..1000000))):
    show(plot(prime_pi,0,N,color='red')+plot(x/(log(x)-1),5,N,color='blue'))
```

What part of the code is producing the slider?

7. Try to figure out what the following Sage Interact will do. Then type it in and try it.

```python
x0 = 0
f = sin(x)*e^(-x)
p = plot(f,-1,5)
dot = point((x0,f(x=x0)),pointsize=80,color='red')
@interact
def i_taylor(order=(5,(1..12))):
    ft = f.taylor(x,x0,order)
    print "The function is \( %s \) \( %f \)"
    print "The order-%s Taylor series at x=%s is \( %s \)\( %f \)"
    pt = plot(ft,-1, 5, color='green')
    show(dot + p + pt, ymin = -.5, ymax = 1)
```

Can you modify this Interact so that the user can enter any desired function? (Some features of the above code depend on facts about the function \( f(x) = \sin(x)e^{-x} \). What else might you need to modify to handle other functions?)

8. If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Write a program to find the sum of all the multiples of 3 or 5 below 1000.

9. When \( n = 0 \), \( n^2 - 79n + 1601 \) is 1601—which is prime. When \( n = 1 \), \( n^2 - 79n + 1601 \) is 1523—which is prime. Find the smallest value of \( n \) where \( n^2 - 79n + 1601 \) is not prime.