LARSON—MATH 255–CLASSROOM WORKSHEET 19
Files—Riemann Integration.

1. (a) Start the Chrome browser.
   (b) Go to `http://cocalc.com`
   (c) Login using **your VCU email address** .
   (d) Click on our class Project.
   (e) Click “New”, then “Worksheets”, then call it **c19**.
   (f) For each problem number, label it in the Sage cell where the work is. So for Problem 2, the first line of the cell should be `#Problem 2`.

Files

2. Now it is the case on any larger program that you will want to use functions you have previously defined. These are called *tools*. Instead of copying and pasting from your old code. You can save them as *files* and load them as needed.

   (a) Click “New”. Type `heads_from_n_flips.sage` and then click “file”. (You are making a **.sage** file *not* our usual Sage Worksheet file. These are regular text files that are loaded as Python files plus some *preprocessing*).
   (b) Define the function:

```
def heads_from_n_flips(n):
    heads=0
    for i in [1..n]:
        if random() < 0.5:
            heads=heads+1
    return heads
```

   (c) Click “Save” and then go back to your **c19** worksheet.
   (d) Type `load("heads_from_n_flips.sage")` and evaluate.
   (e) Now try `heads_from_n_flips(100)` a few times. You never need to write this function again. You have a tool!

Riemann Integration

Given a continuous function $f(x)$ on an interval $[a, b]$ we want to find the *area* between the curve, the x-axis and the lines $y = a$ and $y = b$. One way to do this is to use the Fundamental Theorem of Calculus and integrate. Unfortunately, it is difficult to find anti-derivatives for many (most) functions. So we need a different approach to get at least an approximate integral.

One way to do this is to slice up $[a, b]$ into n equal-sized intervals $[a_0, a_1]$, $[a_1, a_2]$,. . . , $[a_n, a_{n+1}]$ (where $a_1 = a$ and $a_{n+1} = b$), pick a point c_i from each interval $[a_i, a_{i+1}]$ and compute the area $f(c_i) \cdot \Delta$ of a rectangle, where Δ is the interval length $a_{i+1} - a_i$. There are different ways to pick the c_i's. You could pick the leftmost point of the interval, the midpoint, the rightmost point, or even a random point.

The *Riemann Integral* is defined to be the *limit* of these area approximations as n goes to infinity of this quantity.

Here is a function `leftpoint_riemann(f,a,b,n)` which computes the leftpoint Riemann sums for n equal intervals.

```
def leftpoint_riemann(f,a,b,n):
    area=0
    Delta=(b-a)/n
    for i in [0..(n-1)]:
        leftpoint=a+i*Delta
        area=area+f(leftpoint)*Delta
    return area
```

3. Given a continuous function f(x) on [a,b], define a function `rightpoint_riemann(f,a,b,n)` which computes the rightpoint Riemann sums for n equal intervals.

4. Find the values of `rightpoint_riemann(f,a,b,n)`for f(x)=x**2 on [0,3] with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. Compare with your results for `leftpoint_riemann(f,a,b,n)`.

5. **Extra: Learn more Python!**. If you have extra classtime, use it to learn more Python. Go to Codecademy (`codeacademy.com`), sign up for a free account, and do the *Learn Python 2* tutorial `https://www.codecademy.com/learn/learn-python`. (This one is totally free—and useful.)

Getting your classwork recorded

When you are done, before you leave class...

(a) Click the "Make pdf" (Adobe symbol) icon and make a pdf of this worksheet. (If Cocalc hangs, click the printer icon, then "Open", then print or make a pdf using your browser).

(b) Send me an email with an informative header like "Math 255—c19 worksheet attached" (so that it will be properly recorded).

(c) Remember to attach today's classroom worksheet!