Debugging

Given a continuous function $f(x)$ on an interval $[a, b]$ we want to find the area between the curve, the x-axis and the lines $y = a$ and $y = b$.

One way to do this is to slice up $[a, b]$ into $n$ equal-sized intervals $[a_0, a_1], [a_1, a_2], \ldots, [a_n, a_{n+1}]$ (where $a_1 = a$ and $a_{n+1} = b$), pick a point $c_i$ from each interval $[a_i, a_{i+1}]$ and compute the area $f(c_i) \cdot \Delta$ of a rectangle, where $\Delta$ is the interval length $a_{i+1} - a_i$.

There are different ways to pick the $c_i$'s. You could pick the leftmost point of the interval, the midpoint, the rightmost point, or even a random point.

The Riemann Integral is defined to be the limit of these area approximations as $n$ goes to infinity of this quantity.

2. Type in the function leftpoint_riemann($f, a, b, n$) which computes the leftpoint Riemann sums for $n$ equal intervals.

```python
def leftpoint_riemann(f, a, b, n):
    area=0
    Delta=(b-a)/n
    for i in [0..n):
        leftpoint=a+i*Delta
        newarea=f(leftpoint)*Delta
        area=area+newarea
    return area
```

3. Type in the function rightpoint_riemann($f, a, b, n$) which computes the rightpoint Riemann sums for $n$ equal intervals.

```python
def rightpoint_riemann(f, a, b, n):
    area=0
    Delta=(b-a)/n
    for i in [0..n):
        rightpoint=b-i*Delta
        newarea=f(rightpoint)*Delta
        area=area+newarea
    return area
```
4. Find the value of $\text{leftpoint}_\text{riemann}(f,a,b,n)$ for $f(x)=x^2$ on $[0,3]$ with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. Here you are making the intervals smaller and smaller, giving a better and better approximation.

5. Find the values of $\text{rightpoint}_\text{riemann}(f,a,b,n)$ for $f(x)=x^2$ on $[0,3]$ with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. Compare with your results for $\text{leftpoint}_\text{riemann}(f,a,b,n)$. What do you notice???

6. Now let's add print statements to get an idea of what these programs are doing.

```python
def leftpoint_riemann(f,a,b,n):
    area=0
    Delta=(b-a)/n
    for i in [0..n]:
        leftpoint=a+i*Delta
        newarea=f(leftpoint)*Delta
        area=area+newarea
        print "i,leftpoint,newarea are {},{},{}".format(i,leftpoint,newarea)
    return area

def rightpoint_riemann(f,a,b,n):
    area=0
    Delta=(b-a)/n
    for i in [0..n]:
        rightpoint=b-i*Delta
        newarea=f(rightpoint)*Delta
        area=area+newarea
        print "i,rightpoint,newarea are {},{},{}".format(i,rightpoint,newarea)
    return area
```

Can you figure out why these two programs give the same answers? If not, can you think of any more print statements to add that would be helpful?

7. After you figure out why these programs give the same answer, adjust your code so we are calculating what we want. Then recalculate the values of $\text{leftpoint}_\text{riemann}()$ and $\text{rightpoint}_\text{riemann}()$ for $f(x)=x^2$ on $[0,3]$ with $n = 2$, $n = 5$, $n = 10$ and $n = 100$. 