Matrices.

1. Log in to your Sage/Cocalc account.
   (a) Start the Chrome browser.
   (b) Go to http://cocalc.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it c09, then click “Sage Worksheet”.

2. is_prime() is a built-in Sage function which tests if an integer is prime. Try is_prime(5), is_prime(6), is_prime(47), and is_prime(34567)

3. Let L=[1..50]. Now use is_prime and either filter or a list comprehension to filter the primes from list L.

4. Define a function foo(x,y) that takes real numbers x and y and returns 3x + y.

5. Alternatively you could define and name an anonymous function:

   \[ \text{foo} = \lambda x, y: 3 \times x + y \]

   Now try foo(4,5). Note that you never needed to define “y” as a variable.

Sets

6. Let \( L = [2,3,3,3,2,1,8,6,3] \) and \( A=\text{Set}(L) \). Evaluate A. What happened?

7. Find how many elements set A has with A.cardinality().

8. Try: 8 in A. What do you get? Try: 9 in A.

9. Let \( B = \text{Set}([8,6,17,-4,20, -2 ]) \). Try A.union(B). What do you get?

10. Try: A.intersection(B).
11. Let \( R = \{1, 2, 3\} \) and \( S = \{2, 3, 4\} \). What is the set \( R - S \)? Try \( R . d i f f e r e n c e ( S ) \) and \( S . d i f f e r e n c e ( R ) \).

**Matrices**

12. We can represent the system of linear equations
\[
\begin{align*}
2x + y &= 5 \\
x + 3y &= 7
\end{align*}
\]

with the matrix \( A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & 7 \end{bmatrix} \).

Enter this in Sage using: \( A = m a t r i x ( 2, 3, [2, 1, 5, 1, 3, 7] ) \).

Use \( A . r r e f () \) to find a matrix that represents an equivalent system in *row-reduced echelon form*.

13. Consider the system:
\[
\begin{align*}
x + 3y &= 5 \\
x + 3y &= 7
\end{align*}
\]

Find a matrix that represents this system, find the row-reduced echelon form of this matrix, rewrite this as an equivalent system of linear equations and interpret.

14. Consider the system:
\[
\begin{align*}
x + y &= 5 \\
2x + 2y &= 10
\end{align*}
\]

Find a matrix that represents this system, find the row-reduced echelon form of this matrix, rewrite this as an equivalent system of linear equations and interpret.

15. Let \( A = m a t r i x ( 2, 2, [1, 2, 3, 4] ) \), and \( b = v e c t o r ([5, 6]) \). Solve the matrix equation \( A \hat{x} = \hat{b} \) using \( A . s o l v e _ r i g h t ( b ) \).

16. **Challenge.** 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?