1. Log in to your Sage/Cocalc account.

   (a) Start the Chrome browser.
   (b) Go to http://cocalc.com and sign in.
   (c) You should see an existing Project for our class. Click on that.
   (d) Click “New”, call it b02, then click “Sage Worksheet”.
   (e) For each problem, number the cell in your worksheet where the code is. So for Problem 7 below, start the cell with the comment #Problem 7.
   (f) Write only the answers below.
   (g) Make a pdf of your Sage b02 worksheet (click the “make pdf”) button, and email me the work that shows how you computed the answers. Title the email: “Work for 255 bonus worksheet”.
   (h) Do your best. Its OK if you don’t get every answer—but you should have a majority.

2. If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Write a program to find the sum of all the multiples of 3 or 5 below 1000.

3. When \( n = 0 \), \( n^2 - 79n + 1601 \) is 1601—which is prime. When \( n = 1 \), \( n^2 - 79n + 1601 \) is 1523—which is prime. Find the smallest value of \( n \) where \( n^2 - 79n + 1601 \) is not prime.

4. The prime factors of 13195 are 5, 7, 13 and 29. Write a function \texttt{largest_prime_factor(n)} which returns the largest prime factor of a given integer \( n \). What is the largest prime factor of the number 600851475143?

5. 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?
6. The sum of the reciprocals of the positive integers

\[ \sum_{n=1}^{\infty} \frac{1}{n} \]

diverges (that is, the sum goes to infinity).

(a) Find the smallest integer \( m \) so that \( \sum_{n=1}^{m} \frac{1}{n} \) is at least 2.

(b) Find the smallest integer \( m \) so that \( \sum_{n=1}^{m} \frac{1}{n} \) is at least 3.

(c) Find the smallest integer \( m \) so that \( \sum_{n=1}^{m} \frac{1}{n} \) is at least 4.

7. Let \( p_1, p_2, \ldots, p_k \) be a list of any \( k \) primes. The product \( p \) of these primes plus one is

\[ p = p_1 \cdot p_2 \cdot \ldots \cdot p_k + 1 \]

\( p \) is either a prime (different from each of these \( k \) primes) or it has a prime factor also different from each of these. (This implies there are infinitely many primes). Write a program to find the smallest prime number \( q \) no more than \( p \) and different from each of \( p_1, p_2, \ldots, p_k \).

8. The Fibonacci sequence \( F_n \) is defined as follows \( F_0 = 0, F_1 = 1 \) and \( F_n = F_{n-1} + F_{n-2} \) for \( n > 1 \). What is the first term in the Fibonacci sequence to contain 1000 digits?

9. Find the sum of the even Fibonacci numbers less than four million.

10. By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13. What is the 10,001st prime number?

11. \( n! \) means \( n \times (n-1) \times \ldots \times 3 \times 2 \times 1 \). For example, \( 10! = 10 \times 9 \times \ldots \times 3 \times 2 \times 1 = 3628800 \), and the sum of the digits in the number 10! is \( 3 + 6 + 2 + 8 + 8 + 0 + 0 = 27 \). Find the sum of the digits in the number 100!

12. Do the problem at [http://projecteuler.net/problem=8](http://projecteuler.net/problem=8).