Last name _____

First name _____

LARSON—MATH 255–CLASSROOM WORKSHEET 17
Files & Interacts.

1. Log in to your Sage/Cocalc account.

   (a) Start the Chrome browser.

   (b) Go to `http://cocalc.com` and sign in.

   (c) You should see an existing Project for our class. Click on that.

   (d) Click "New", call it **c17**, then click "Sage Worksheet".

   (e) For each problem number, label it in the Sage cell where the work is. So for Problem 1, the first line of the cell should be `#Problem 1`.

   (f) When you are finished with the worksheet, click "make pdf", email me the pdf (at `clarson@vcu.edu`, with a header that says **Math 255 c17 worksheet attached**).

   There is a collection of examples of Sage INTERACTS at `http://wiki.sagemath.org/interact/`. Let's look at a few of these examples to see the kinds of things you can do with Sage.

   Warm-Ups

2. Here is a simple Sage INTERACT with *default* values for a function f to be graphed, and the interval (a, b) that f will be graphed on. The Interact function is named "_", which is standard—as we will never refer to this function by name.

```
@interact
def _(f=x^2, a=-3, b=3):
    show(plot(f,(x,a,b)))
```

3. Now let's make this fancy with some *options*.

```
@interact
def _(f=input_box(x^2,width=20),
    axes=True,
    fill=True,
    zoom=range_slider(-3,3,default=(-3,3))):
    show(plot(f,(x,zoom[0], zoom[1]), axes=axes,fill=fill))
```

Eigenvalues

4. Try `M=identity_matrix(3)`. Evaluate M to see the entries. Change the upper right corner entry to 3. Use `M.` and TAB to find the eigenvalues and eigenvectors of M

5. Here's an INTERACT with an *update* button.

```
@interact
def _(m=('matrix', identity_matrix(2)), auto_update=False):
    print m.eigenvalues()
```

   Try different values in the matrix and then and then click the update box. Now make a 3×3 matrix.

Problems

6. 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

7. If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Write a program to find the sum of all the multiples of 3 or 5 below 1000.

8. When $n = 0$, $n^2 - 79n + 1601$ is 1601—which is prime. When $n = 1$, $n^2 - 79n + 1601$ is 1523—which is prime. Find the smallest value of n where $n^2 - 79n + 1601$ is *not* prime.

9. The prime factors of 13195 are 5, 7, 13 and 29. Write a function `largest_prime_factor(n)` which returns the largest prime factor of a given integer n. What is the largest prime factor of the number 600851475143?

   When your function works, copy it to a *file* and save it with filename `largest_prime_factor`. Now you have another tool—that you can load later as needed!

10. The sum of the reciprocals of the positive integers

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

    diverges (that is, the sum goes to infinity).

    (a) Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 2.
    (b) Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 3.
    (c) Find the smallest integer m so that $\sum_{n=1}^{m} \frac{1}{n}$ is at least 4.

11. Let $p_1, p_2, \ldots, p_k$ be a list of any k primes. The product p of these primes plus one is

$$p = p_1 \cdot p_2 \cdot \ldots p_k + 1$$

p is either a prime (different from each of these k primes) or it has a prime factor also different from each of these. (This implies there are infinitely many primes). Write a program to find the smallest prime number q no more than p and different from each of $p_1, p_2, \ldots, p_k$.

12. The Fibonacci sequence F_n is defined as follows $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 1$.

Here is an iterative function `fibonacci2(n)` that computes the nth Fibonacci number.

```
def fibonacci2(n):
    F=[0,1]
    for i in [2..n]:
        F.append(F[i-1]+F[i-2])
    return F[n]
```

What is the first term in the Fibonacci sequence to contain 1000 digits?