Ultrafilters over \( \mathbb{N} \) are certain combinatorial objects constructed using Zorn’s Lemma. Given a sequence \( \mathcal{G} = \langle G_n : n \in \mathbb{N} \rangle \) of, say, finite graphs, an ultrafilter can be used to construct an ultraproduct of \( \mathcal{G} \), and if \( P \) is a first order property such that all but finitely many of the \( G_n \)’s have property \( P \), then the ultraproduct also has property \( P \). Although such an ultraproduct is typically very large—in fact uncountably infinite—it can still be used to provide information about the behavior of random finite graphs; for example, to prove the famous 0-1 laws for the limiting behavior of finite random graphs. All of these are classic results due to Fagin and Glebskii-Kogan-Liagonkii-Talanov independently.

This talk is a preview to one of the topics to be covered in the Spring 2017 special topics course Filters, ultrafilters, and applications (Math 591.002).