

LARSON—MATH 310—WORKSHEET 12

Vector Spaces V. Solutions of a homogeneous system of linear equations.

Let  $V$  be the set of solutions of a homogenous system of  $m$  linear equations in  $n$  variables. Define “addition” and “scalar multiplication” in the obvious way.

- Let  $\vec{v}, \vec{u} \in V$ .
  - What do  $\vec{v}$  and  $\vec{u}$  look like?
  - Find  $\vec{v} + \vec{u}$ . Is  $\vec{v} + \vec{u}$  in  $V$ ?
- Find  $\vec{u} + \vec{v}$ . Is  $\vec{v} + \vec{u} = \vec{u} + \vec{v}$ ?
- Let  $\vec{v}, \vec{u}, \vec{w} \in V$ .
  - Find  $(\vec{v} + \vec{u}) + \vec{w}$ .
  - Find  $\vec{v} + (\vec{u} + \vec{w})$ .
  - Does  $(\vec{v} + \vec{u}) + \vec{w} = \vec{v} + (\vec{u} + \vec{w})$ ?
- Is there a  $\vec{0}$  in  $V$  such that  $\vec{v} + \vec{0} = \vec{v}$  for every  $\vec{v}$  in  $V$ ?
- Let  $\vec{v}$  be in  $V$ . Is there a  $\vec{w}$  in  $V$  so that  $\vec{v} + \vec{w} = \vec{0}$ ? Explain. (What does  $\vec{w}$  look like?)
- Let  $r \in \mathbb{R}$ . Is  $r\vec{v}$  in  $V$ ? Explain.
- Let  $r, s \in \mathbb{R}$ .
  - Find  $(r + s)\vec{v}$ .

- Find  $s\vec{v}$ .
- Find  $r\vec{v} + s\vec{v}$ .
- Does  $(r + s)\vec{v} = r\vec{v} + s\vec{v}$ ?

8. Let  $r \in \mathbb{R}$ ,  $\vec{v}, \vec{w} \in V$ .

- Find  $r(\vec{v} + \vec{w})$ .
- Find  $r\vec{v}$ .
- Find  $r\vec{w}$ .
- Find  $r\vec{v} + r\vec{w}$ .
- Does  $r(\vec{v} + \vec{w}) = r\vec{v} + r\vec{w}$ ?

9. Let  $r, s \in \mathbb{R}$ .

- Find  $(rs)\vec{v}$ .
- Find  $s\vec{v}$ .
- Find  $r(s\vec{v})$ .
- Does  $(rs)\vec{v} = r(s\vec{v})$ ?

10. Let  $\vec{v} \in V$ .

- Find  $1\vec{v}$ .
- Does  $1\vec{v} = \vec{v}$ ?