

## MAMLS at VCU – Abstracts

All talks will take place in Temple Building, Room 1165 located at 901 W Main St, Richmond, VA 23284. Breakfast and coffee will be provided just outside of Room 1165.

==Sat., April 1==

---

Open and clopen determinacy for proper class games

**Joel David Hamkins** (City University of New York)

9:30 - 10:30

The principle of open determinacy for class games — two-player games of perfect information with plays of length  $\omega\omega$ , where the moves are chosen from a possibly proper class, such as games on the ordinals — is not provable in Zermelo-Fraenkel set theory ZFC or Gdel-Bernays set theory GBC, if these theories are consistent, because provably in ZFC there is a definable open proper class game with no definable winning strategy. In fact, the principle of open determinacy and even merely clopen determinacy for class games implies  $\text{Con}(\text{ZFC})$  and iterated instances  $\text{Con}(\text{Con}(\text{ZFC}))$  and more, because it implies that there is a satisfaction class for first-order truth, and indeed a transfinite tower of truth predicates  $\text{Tr}_\alpha$  for iterated truth-about-truth, relative to any class parameter. This is perhaps explained, in light of the Tarskian recursive definition of truth, by the more general fact that the principle of clopen determinacy is exactly equivalent over GBC to the principle of elementary transfinite recursion ETR over well-founded class relations. Meanwhile, the principle of open determinacy for class games is provable in the stronger theory  $\text{GBC} + \Pi_1^1$ -comprehension, a proper fragment of Kelley-Morse set theory KM. New work by Hachtman and Sato, respectively has clarified the separation of clopen and open determinacy for class games.

This is joint work with Victoria Gitman. Comments and questions can be made on the speakers blog at <http://jdh.hamkins.org/open-and-clopen-determinacy-for-proper-class-games-vcu-mamls-april-2017/>.

---

On the distance between HOD and V

**Omer Ben-Neria** (University of California at Los Angeles)

10:45 - 11:45

The pursuit of better understanding the universe of set theory V motivated an extensive study of definable inner models M whose goal is to serve as good approximations to V. A common property of these inner models is that they are contained in HOD, the universe of hereditarily ordinal definable sets. Motivated by the question of how “close” HOD is to V, we consider various related forcing methods and survey known and new results. This is a joint work with Spencer Unger.

---

Iteration with non-stationary support

**James Cummings** (Carnegie Mellon University)

12:00 - 1:00

We present some applications of iterated forcing with non-stationary support to problems about tall cardinals (joint work with Arthur Apter)

---

Simultaneous stationary reflection and failure of SCH

**Dima Sinapova** (University of Illinois at Chicago)

2:30 - 3:30

We will show that it is consistent to have finite simultaneous stationary reflection at  $\kappa^+$  with not SCH at  $\kappa$ . This extends a result of Assaf Sharon. We will also present an abstract approach of iterating Prikry type forcing and use it to bring our construction down to  $\aleph_\omega$ . This is joint work with Assaf Rinot.

---

The approachability ideal without a maximal set

**John Krueger** (University of North Texas)

3:45 - 4:45

I will discuss my proof of the consistency that the approachability ideal on  $\omega_2$  does not have a maximal set modulo clubs. The forcing construction involves many of the ideas used by Mitchell in his proof of the consistency that the approachability ideal can be trivial, including side conditions, strongly generic conditions, the approachability property, and greatly Mahlo cardinals. It also includes new ideas, such as forcing a partial square sequence with finite conditions, a side condition product forcing, and a technical weakening of Mitchell's notion of strongly generic tidy conditions.

---

The strategic transition

**Hugh Woodin** (Harvard University)

5:00 - 6:00

There is a fairly general argument which predicts that the hierarchy of nonstrategic-extender models cannot (provably) reach the level of yielding a weak extender model for the supercompactness of some cardinal. These arguments can now be sharpened to identify a very precise threshold where one must transition from the hierarchy of non-strategic-extender models to the hierarchy of strategic-extender models. This involves connecting with the problem of generic conditional absoluteness for  $\Sigma_2^2$ -sentences.

---

**==Sun., April 2==**

Global Chang's Conjecture

**Monroe Eskew** (Virginia Commonwealth University)

9:30 - 10:30

I present results from a recent paper with Yair Hayut. We substantially reduce the upper bound on consistency strength for many instances of Chang's Conjecture. For example,  $(\omega_4, \omega_3) \rightarrow (\omega_2, \omega_1)$  is shown to be consistent relative to a  $(+2)$ -subcompact cardinal. From a huge cardinal, we obtain a model in which for all regular  $\kappa$  and all  $\mu < \kappa$ ,  $(\kappa^+, \kappa) \rightarrow (\mu^+, \mu)$ , answering a question of Foreman.

---

Virtual set theory and generic Vopěnka's principle

**Victoria Gitman** (City University of New York)

10:45 - 11:45

Given a set theoretic property  $\mathcal{P}$  characterized by the existence of elementary embeddings between some first-order structures, we will say that  $\mathcal{P}$  holds *virtually* if the embeddings of structures from  $V$  characterizing  $\mathcal{P}$  exist in the generic multiverse of  $V$ . The idea of studying virtual properties was introduced by Ralf Schindler, growing out of his work on remarkable cardinals. Large cardinals are the obvious candidates for virtualization, but the idea can be fruitfully applied to other properties such as forcing axioms and Vopěnka's Principle. I will talk about joint work with Schindler on the virtual versions of larger large cardinals such as extendibles,  $n$ -huge, and rank-into-rank cardinals. The virtual versions of these large cardinals form a hierarchy, mirroring that of their actual counterparts, which is consistent with  $V = L$ . Virtual large cardinals have proven useful for measuring consistency strength of other virtual properties as well as assertions arising from completely different contexts. It is even possible to have (consistent) virtual versions of inconsistent large cardinals: for instance, a forcing extension can have elementary embeddings  $j : V_{\lambda+2}^V \rightarrow V_{\lambda+2}^V$ . I will also talk about *Generic Vopěnka's Principle*, a virtual version of Vopěnka's Principle. Bagaria showed that Vopěnka's Principle holds if and only if for every  $n \in \omega$ , there is a proper class of  $C^{(n)}$ -extendible cardinals. With Bagaria and Schindler, we showed that Generic Vopěnka's Principle is consistent with  $V = L$  and that it is equiconsistent with the assertion that for every  $n \in \omega$ , there is a proper class of virtually  $C^{(n)}$ -extendible cardinals. With Hamkins, we show that, in contrast with Vopěnka's Principle, a model of Generic Vopěnka's Principle need not have any virtually  $C^{(n)}$ -extendible cardinals.