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Test 3

MATH 200, Section 3

June 11, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (12 points) This problem concerns the equation $x^2 + xy - y^2 = 1$. $y = f(x)$

(a) Find y'.

$$\frac{d}{dx}\left[x^2 + xy - y^2\right] = \frac{d}{dx}[1]$$

$$2x + 1y + xy' - 2yy' = 0$$

$$xy' - 2yy' = -2x - y$$

$$y'(x - 2y) = -2x - y$$

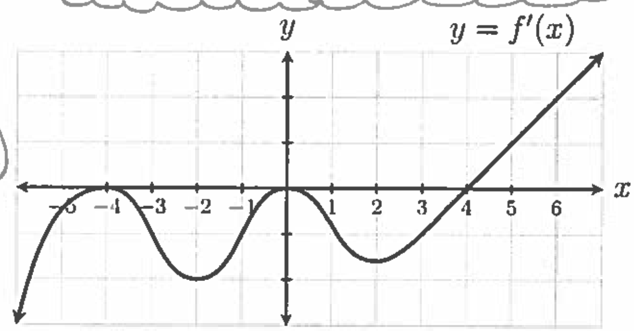
$$\boxed{y' = \frac{-2x - y}{x - 2y}}$$

(b) Use part (a) to find the slope of the tangent line to the graph of $x^2+xy-y^2=1$ at the point $(2, 3)$.

$$y'\Big|_{(x,y)=(2,3)} = \frac{-2\cdot 2 - 3}{2 - 2\cdot 3} = \frac{-7}{-4} = \boxed{\frac{7}{4}}$$

2. (12 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

(a) State the critical points of f. $\boxed{-4, 0, 4}$ ← Because they make $f'(x) = 0$



(b) State the interval(s) on which f increases.

$\boxed{(4, \infty)}$ (because $f'(x) > 0$ there)

(c) State the interval(s) on which f decreases.

$\boxed{(-\infty, -4) \cup (-4, 0) \cup (0, 4)}$

(d) State the interval(s) on which f is concave down.

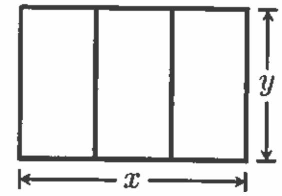
$\boxed{(-4, -2) \text{ and } (0, 2)}$ because that's where $f'(x)$ decreases, so $f''(x) < 0$

3. (10 points) Is the equation $\int \ln(x)\, dx = x\ln(x) - x + C$ true or false? Explain.

Let's check: $\frac{d}{dx}\left[x\ln(x) - x + C\right] = 1\cdot\ln(x) + x\frac{1}{x} - 1 = \ln(x)$

We got the integrand so YES this is TRUE.

4. (12 points) You have 200 feet of chain link fence to enclose three rectangular regions, as shown below. Find the dimensions x and y that maximize the enclosed area.



Need to maximize area $= xy$

$= x\left(50 - \frac{x}{2}\right)$

$= 50x - \frac{x^2}{2}$

Constraint:

$2x + 4y = 200$

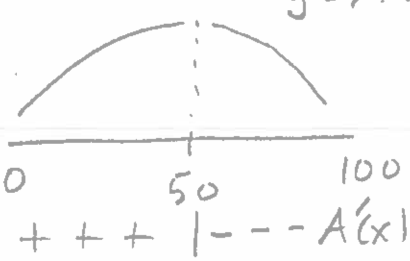
$4y = 200 - 2x$

$\boxed{y = 50 - \frac{x}{2}}$

So we need to find the x giving a global maximum of Area $=$

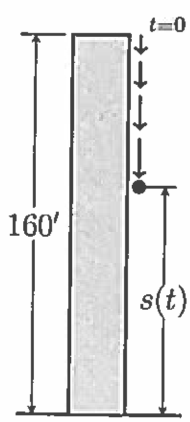
$A(x) = 50x - \frac{x^2}{2}$ on $(0, 100)$

$A'(x) = 50 - x = 0$ $x = 50$ is critical point



<u>Answer</u> Greatest area when $x = 50$ and $y = 50 - \frac{50}{2} = 25$

5. (12 points) An object is propelled straight down from atop a 160-foot-high tower at time $t = 0$ seconds. At time t seconds its height is $s(t) = 160 - 16t^2 - 48t$ feet.
Use algebra and calculus to find the object's velocity on impact with the ground.



Object hits ground when $160 - 16t^2 - 48t = 0$

$16(10 - t^2 - 3t) = 0$

$-16(t^2 + 3t - 10) = 0$

$16(t-2)(t+5) = 0$

$t = 2$ $\quad$ $t = -5$

Thus object hits ground when $t = 2$ sec and $t = -5$ but disregard -5 sec because we're not considering negative time.

Velocity $= v(t) = s'(t) = 32t - 48$

Thus velocity on impact is $s'(2) = -32 \cdot 2 - 48 = 112 \frac{ft}{sec}$

6. (21 points) Find the limits.

(a) $\lim_{x\to 0} \frac{\cos(x) - 5x - 1}{2x} = \lim_{x\to 0} \frac{-\sin(x) - 5}{2} = \frac{-\sin(0)-5}{2} = \boxed{\frac{-5}{2}}$

(form $\frac{0}{0}$)

(b) $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = \boxed{0}$

(form $\infty \cdot 0$) (form $\frac{\infty}{\infty}$) (denominator goes to ∞)

(c) $\lim_{x\to\infty} \big(\ln(x) - \ln(x+1)\big) = \lim_{x\to\infty} \left(\ln\left(\frac{x}{x+1}\right)\right)$

$= \ln\left(\lim_{x\to\infty} \frac{x}{x+1}\right) = \ln(1) = \boxed{0}$

7. (21 points) Find the integrals.

(a) $\int \left(x^6 + \frac{1}{x} + \frac{1}{x^3}\right) dx = \int \left(x^6 + \frac{1}{x} + x^{-3}\right) dx = \frac{1}{7}x^7 + \ln|x| + \frac{1}{-3+1}x^{-3+1} + C$

$= \boxed{\frac{x^7}{7} + \ln|x| - \frac{1}{2x^2} + C}$

(b) $\int (x + \sin(x) - 1)\, dx = \boxed{\frac{1}{2}x^2 - \cos(x) - x + C}$

(c) $\int \left(e^x + \frac{1}{1+x^2}\right) dx = \boxed{e^x + \tan^{-1}(x) + C}$