Name: _____

Test 3

MATH 200, Section 3

June 11, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (12 points) This problem concerns the equation $x^2 + xy - y^2 = 1$.

   (a) Find y'.

   (b) Use part (a) to find the slope of the tangent line to the graph of $x^2+xy-y^2=1$ at the point $(2, 3)$.

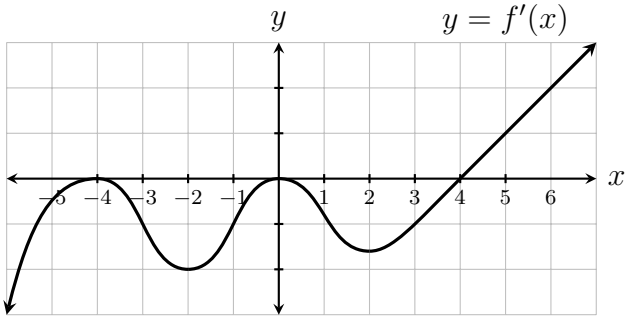
2. (12 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

   (a) State the critical points of f.

   (b) State the interval(s) on which f increases.

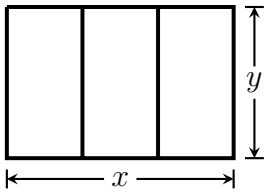
   (c) State the interval(s) on which f decreases.

   (d) State the interval(s) on which f is concave down.

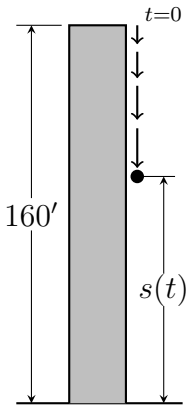


3. (10 points) Is the equation $\int \ln(x)\,dx = x\ln(x) - x + C$ true or false? Explain.

4. (12 points) You have 200 feet of chain link fence to enclose three rectangular regions, as shown below. Find the dimensions x and y that maximize the enclosed area.



5. (12 points) An object is propelled straight down from atop a 160-foot-high tower at time $t = 0$ seconds. At time t seconds its height is $s(t) = 160 - 16t^2 - 48t$ feet.
Use algebra and calculus to find the object's velocity on impact with the ground.



6. (21 points) Find the limits.

(a) $\lim_{x\to 0} \dfrac{\cos(x) - 5x - 1}{2x} =$

(b) $\lim_{x\to \infty} xe^{-x} =$

(c) $\lim_{x\to \infty} \big(\ln(x) - \ln(x+1)\big) =$

7. (21 points) Find the integrals.

(a) $\displaystyle\int \left(x^6 + \frac{1}{x} + \frac{1}{x^3}\right) dx =$

(b) $\displaystyle\int \big(x + \sin(x) - 1\big)\, dx =$

(c) $\displaystyle\int \left(e^x + \frac{1}{1+x^2}\right) dx =$