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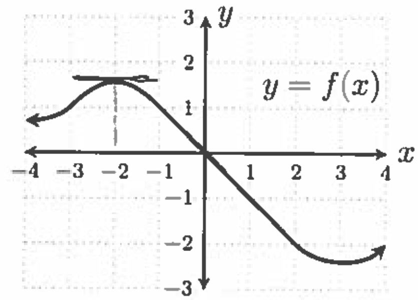
TEST 2

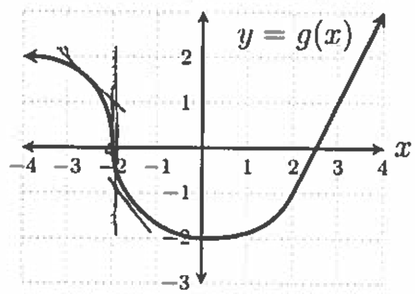
MATH 200, SECTION 3

June 4, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (15 points) Answer the questions about the functions graphed below.





(a) $f'(-2) =$ $\boxed{0}$

(b) $f'(0) =$ $\boxed{-1}$

(c) $\lim_{x \to -2} g'(x) =$ $\boxed{-\infty}$

(d) If $h(x) = f(x)g(x)$, then $h'(0) =$ $f'(0)g(0) + f(0)g'(0)$

$= (-1)(-2) + 0 \cdot 0 = \boxed{2}$

(e) If $h(x) = f(g(x))$, then $h'(3) =$ $f'(g(3))g'(3)$

$= f'(1) \cdot 2 = (-1)(2)$

$= \boxed{-2}$

2. (8 points) Find the derivatives of the following functions.

(a) $f(x) = x^4 - 3x + \pi^2$ $\quad f'(x) = \boxed{4x^3 - 3}$

(b) $f(x) = \sin^{-1}(x)$ $\quad f'(x) = \boxed{\frac{1}{\sqrt{1-x^2}}}$

(c) $f(x) = e^{-x}$ $\quad f'(x) = \boxed{-e^{-x}}$

(d) $f(x) = \sin(\pi x)$ $\quad f'(x) = \boxed{\cos(\pi x) \cdot \pi}$

3. (10 points) Find the equation of the tangent line to the graph of $y = \tan(x)$ at the point where $x = \pi/4$.

Point: $\left(\frac{\pi}{4}, \tan\left(\frac{\pi}{4}\right)\right) = \left(\frac{\pi}{4}, 1\right)$

Slope: $m = f'(\pi/4) = \sec^2(\pi/4) = \frac{1}{\cos^2(\pi/4)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = \boxed{2}$

Point-Slope formula: $y - y_0 = m(x - x_0)$

$y - 1 = 2(x - \pi/4)$

$$\boxed{y = 2x - \frac{\pi}{2} + 1}$$

4. (30 points) Find the derivatives.

(a) $\frac{d}{dx}\left[\sqrt{x^4+x^2+1}\right] = \frac{d}{dx}\left[(x^4+x^2+1)^{\frac{1}{2}}\right] = \frac{1}{2}(x^4+x^2+1)^{-\frac{1}{2}}(4x^3+2x)$

$$= \boxed{\frac{4x^3+2x}{2\sqrt{x^4+x^2+1}}}$$

(b) $\frac{d}{dx}\left[x^2\cos(x^2)\right] = 2x\cos(x^2) + x^2\frac{d}{dx}\left[\cos(x^2)\right]$

$$= 2x\cos(x^2) + x^2(-\sin(x^2)2x)$$

$$= \boxed{2x\cos(x^2) - 2x^3\sin(x^2)}$$

(c) $\frac{d}{dx}\left[\frac{e^x}{x}\right] = \frac{e^x x - e^x \cdot 1}{x^2} = \boxed{\frac{e^x(x-1)}{x^2}}$

(d) $\frac{d}{dx}\left[\frac{1}{\sqrt{3x+1}}\right] = \frac{d}{dx}\left[(3x+1)^{-\frac{1}{2}}\right] = -\frac{1}{2}(3x+1)^{-\frac{1}{2}-1}\cdot 3$

$$= -\frac{3}{2}(3x+1)^{-3/2} = \boxed{\frac{-3}{2\sqrt{3x+1}^{\,3}}}$$

(e) $\frac{d}{dx}\left[\ln(\sec(e^x))\right] = \frac{1}{\sec(e^x)}\frac{d}{dx}\left[\sec(e^x)\right]$

$$= \frac{1}{\sec(e^x)}\sec(e^x)\tan(e^x)e^x$$

$$= \boxed{\tan(e^x)e^x}$$

5. (7 points) $\lim_{h\to 0} \frac{\ln(x+h)-\ln(x)}{h} =$ $\frac{1}{x}$ (Definition of $\frac{d}{dx}[\ln(x)]$)

6. (10 points) Suppose $y = x\ln(x) - x$.

(a) $\frac{dy}{dx} =$ $1\ln(x) + x\frac{1}{x} - 1 = \ln(x) + 1 - 1 = \boxed{\ln(x)}$

(b) $\frac{d^2y}{dx^2} =$ $\frac{d}{dx}\left[\ln(x)\right] = \boxed{\frac{1}{x}}$

(c) $\frac{d^3y}{dx^3} =$ $\frac{d}{dx}\left[\frac{1}{x}\right] = \boxed{\frac{-1}{x^2}}$

7. (10 points) Find all x for which the tangent to $f(x) = \frac{x^2-6x+10}{x-3}$ at $(x, f(x))$ has slope 0.

Need to solve $f'(x) = 0$

$$\frac{(2x-6)(x-3)-(x^2-6x+10)(1)}{(x-3)^2} = 0$$

multiply both sides by $(x-3)^2$

$$(2x-6)(x-3)-(x^2-6x+10) = 0$$

$$2x^2-12x+18-x^2+6x-10 = 0$$

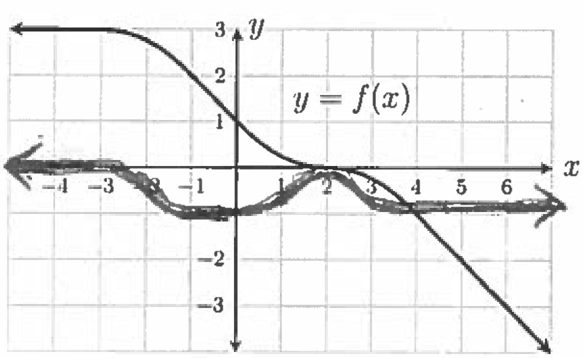
$$x^2-6x+8 = 0$$

$$(x-2)(x-4) = 0$$

$x=2$ $\qquad$ $x=4$

Answer: at $x=2$ and $x=4$

8. (10 points) A function $f(x)$ is graphed below. Sketch the graph of its derivative $f'(x)$.



$y = f'(x)$