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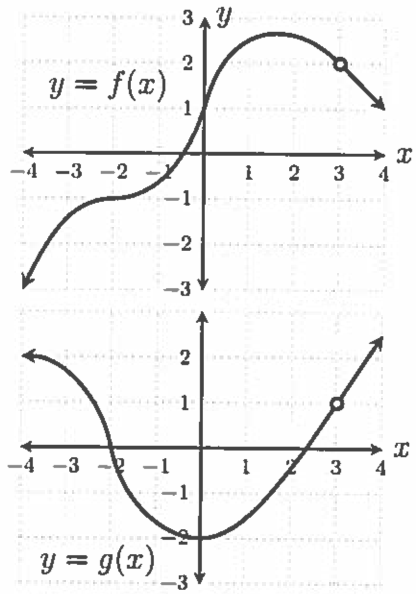
TEST 1

MATH 200, SECTION 3

May 28, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (15 points) Answer the questions about the functions graphed below.



(a) $\lim_{x\to -2} g(x) = \boxed{0}$

(b) $\lim_{x\to -2} \frac{\sin(g(x))}{g(x)} = \boxed{1}$ (because $g(x)$ is approaching 0)

(c) $\lim_{x\to 3} \frac{f(x)}{2+g(x)} = \frac{2}{2+1} = \boxed{\frac{2}{3}}$

(d) $\lim_{x\to 0} f(x)g(x) = 1\cdot(-2) = \boxed{-2}$

(e) $\lim_{x\to 0} f(g(x)) = f\left(\lim_{x\to 0} g(x)\right) = f(-2) = \boxed{-1}$

2. (15 points) Draw the graph of **one** function $f(x)$ meeting **all** of the following conditions.

(a) The domain of f is $(-\infty, 1)\cup(1,\infty)$.

(b) The function f is continuous at all x except $x=-2$, $x=1$ and $x=4$.

(c) $\lim_{x\to 1} f(x) = -\infty$

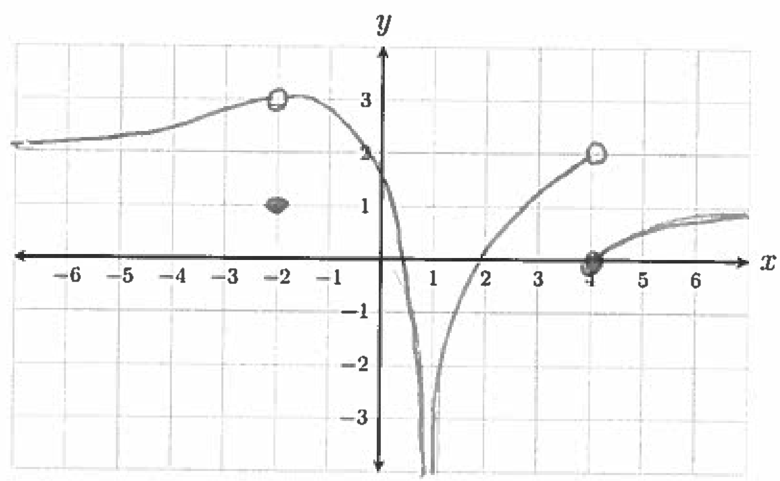
(d) $\lim_{x\to -2} f(x) = 3$

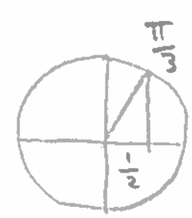
(e) $\lim_{x\to 4^-} f(x) = 2$

(f) $\lim_{x\to 4^+} f(x) = 0$

(g) $\lim_{x\to \infty} f(x) = 1$

(h) $\lim_{x\to -\infty} f(x) = 2$



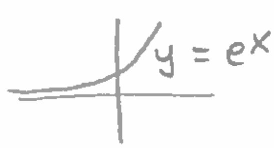


3. (15 points) Find the limits

(a) $\lim_{x\to \pi/3} \cos(x) = \cos(\pi/3) = \boxed{\frac{1}{2}}$

(b) $\lim_{x\to \pi/2} \ln(\sin(x)) = \ln\left(\lim_{x\to \pi/2} \sin(x)\right) = \ln(\sin(\pi/2)) = \ln(1) = \boxed{0}$

(c) $\lim_{x\to -\infty} e^x = \boxed{0}$



4. (30 points) Find the limits

(a) $\displaystyle\lim_{x\to\infty}\frac{x^2+8x-20}{2x^2+2x-12} = \lim_{x\to\infty}\frac{x^2+8x-20}{2x^2+2x-12}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$$= \lim_{x\to\infty}\frac{1+\frac{8}{x}-\frac{20}{x^2}}{2+\frac{2}{x}-\frac{12}{x^2}} = \frac{1+0+0}{2+0+0} = \boxed{\frac{1}{2}}$$

(b) $\displaystyle\lim_{x\to 2}\frac{x^2+8x-20}{2x^2+2x-12} = \lim_{x\to 2}\frac{(x-2)(x+10)}{2(x^2+x-6)}$

$$= \lim_{x\to 2}\frac{\cancel{(x-2)}(x+10)}{2\cancel{(x-2)}(x+3)} = \lim_{x\to 2}\frac{x+10}{2(x+3)} = \frac{2+10}{2(2+3)} = \boxed{\frac{6}{5}}$$

(c) $\displaystyle\lim_{x\to -3^+}\frac{x^2+8x-20}{2x^2+2x-12} = \lim_{x\to -3^+}\frac{(x-2)(x+10)}{2(x-2)(x+3)}$ (same factoring as above)

$$= \lim_{x\to -3^+}\frac{x+10}{2(x+3)} = \boxed{\infty}$$

(approaches 7)

(approaches 0, pos.)

$-3 \leftarrow x$

(d) $\displaystyle\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9}\frac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$$= \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$$

(e) $\displaystyle\lim_{x\to 0}\frac{\cos^2(x)-\cos(x)}{\cos(x)-1} = \lim_{x\to 0}\frac{\cos(x)\cancel{(\cos(x)-1)}}{\cancel{\cos(x)-1}}$

$$= \lim_{x\to 0}\cos(x) = \cos(0) = \boxed{1}$$

5. (10 points) Find the value a such that f is continuous on $(-\infty, \infty)$, where f is defined as

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 2 \\ 5x + a & \text{if } x \geq 2 \end{cases}$$

Note that $f(x) = 3x-2$ on $(-\infty, 2)$ so it is continuous there. Also $f(x) = 5x+a$ on $(2, \infty)$ so it is continuous there. We just need it to be continuous at $x = 2$. For this we need

$$\lim_{x\to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x\to 2^-} (3x-2) = \lim_{x\to 2^+} 5x+a$$

$$3\cdot 2 - 2 = 5\cdot 2 + a \longrightarrow 4 = 10 + a$$

$$\boxed{a = -6}$$

6. (15 points) Use a limit to find the slope of the tangent line to $f(x) = \frac{6}{x}$ at the point $(6, 1)$.

$$m = \lim_{z\to a} \frac{f(z) - f(a)}{z - a}$$

$$= \lim_{z\to 6} \frac{\frac{6}{z} - \frac{6}{6}}{z-6}$$

$$= \lim_{z\to 6} \frac{\frac{6}{z} - 1}{z - 6}$$

$$= \lim_{z\to 6} \frac{\frac{6}{z} - 1}{z-6} \cdot \frac{z}{z}$$

$$= \lim_{z\to 6} \frac{6-z}{(z-6)z} = \lim_{z\to 6} \frac{-\cancel{(z-6)}}{\cancel{(z-6)}z}$$

$$= \lim_{z\to 6} \frac{-1}{z} = \boxed{-\frac{1}{6}}$$