MATH 200 – FINAL EXAM

Name: ____________________

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Directions. Answer the questions in the space provided. This is a closed-notes, closed book exam. No calculators, no computers and no formula sheets.
For numeric answers, give exact, simplified quantities. ($\sqrt{2}$ instead of 2.14, etc.).
Put your final answer in a $\boxed{\text{box}}$ when appropriate. You have three hours.

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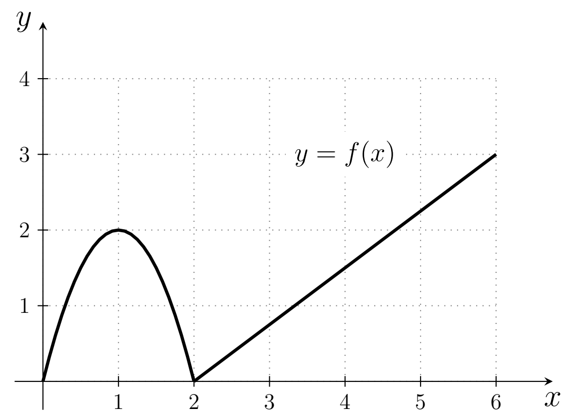
(1) (10 points) Answer the following questions involving the function $f(x)$ graphed below.



(a) $f'(1) =$

(b) $f'(4) =$

(c) $\lim_{x\to 1} \dfrac{3x+1-2f(x)}{x^2-3x-2} =$

(d) $\lim_{x\to 4} \dfrac{2f(x)-3}{x^2-2x-8} =$

(e) $\int_2^6 f(x)\, dx =$

(2) (12 points) Find the limits. Please show work.

(a) $\lim_{x\to\pi} \dfrac{5}{2+\sin(x)} =$

(b) $\lim_{x\to 5} \dfrac{\sqrt{x+4}-3}{x-5} =$

(c) $\lim_{x\to 0} \dfrac{e^{2x}-2x-1}{x^2} =$

(d) $\lim_{x\to\infty} x\tan\left(\frac{3}{x}\right) =$

(3) (18 points) Find the derivatives of the following functions.

(a) $f(x) = 5x^6 + \dfrac{3}{x} - 7\sin(x) + 2$

(b) $f(x) = x^3 \cos(x)$

(c) $p(z) = \dfrac{8z^5}{e^z}$

(d) $f(x) = \ln\left(20x^3 - 7x\right)$

(e) $h(x) = \tan^{-1}\left(5x^2\right)$

(f) $f(x) = (1 + \tan^4(x))^3$

(4) (5 points) Given the equation $y^2 + 9xy = 2x^4$, find y'.

(5) (5 points) Find the value of c for which the following function is continuous at $\frac{\pi}{4}$.

$$f(x) = \begin{cases} \sin^2(x) + c & \text{if } x < \frac{\pi}{4} \\ 1 + \frac{cx}{\pi} & \text{if } x \geq \frac{\pi}{4} \end{cases}$$

(6) (10 points) Determine whether the following statements are **true** or **false**. Explain.

(a) If $f'(c) = 0$ then f must have a local extremum at c.

(b) If $f'(x) < 0$ and $f''(x) > 0$ on an interval, then f is decreasing at an increasing rate.

(c) $\int (x^2 - 1)^2 \, dx = \frac{(x^2 - 1)^3}{3} + C$.

(d) If $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to a^-} f(x) = f(a)$, then f is continuous at a.

(e) If the acceleration of an object is increasing, then its velocity is also increasing.

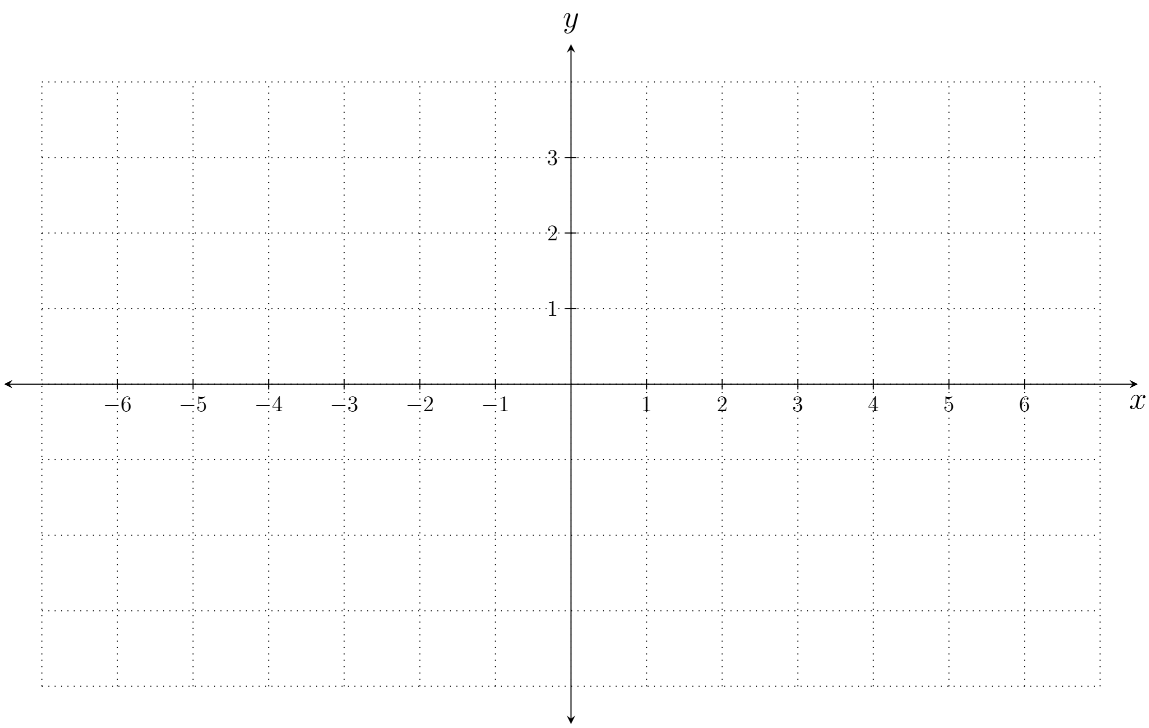
(7) (5 points) Find $\frac{d}{dx} \left[\int_0^{x^2} \frac{1}{(t+2)^3} \, dt \right]$.

(8) (8 points) Draw a graph of $y = f(x)$ meeting **all** of the following conditions.

- f is continuous on $(-\infty, -2) \cup (-2, \infty)$
- f is differentiable on $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$.
- $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = +\infty$
- $\lim_{x \to -\infty} f(x) = +\infty$ and $\lim_{x \to \infty} f(x) = -3$
- f, f' and f'' meet the the conditions in the following table:

x	-3	-2	-1	0
$f(x)$	0	DNE	-1	0
$f'(x)$	0	DNE	0	DNE
$f''(x)$	0	DNE		DNE

- $f'(x) < 0$ on $(-\infty, -2) \cup (-2, -1) \cup (0, \infty)$,
- $f'(x) > 0$ on $(-1, 0)$,
- $f''(x) < 0$ on $(-3, -2)$,
- $f''(x) > 0$ on $(-\infty, -3) \cup (-2, 0) \cup (0, \infty)$.



(9) (4 points) Suppose the derivative of a function $f(x)$ is $f'(x) = (x+3)^2(x-2)(x+1)^2$

(a) Find the intervals where $f(x)$ is increasing/decreasing.

(b) List any local extrema of $f(x)$. Specify whether it is a maximum or minimum.

(10) (8 points) This problem concerns three functions f, g and h.

At $x = 2$, the graph of $y = f(x)$ has tangent line $y = 3x + 4$.

At $x = -1$, the graph of $y = g(x)$ has tangent line $y = -x + 1$.

Suppose $h(x) = f\big(g(x)\big)$.

Answer the following questions using the above information.

(a) $f(2) =$

(b) $f'(2) =$

(c) $g(-1) =$

(d) $g'(-1) =$

(e) $h(-1) =$

(f) $h'(-1) =$

(g) Find the tangent line to the graph of $y = h(x)$ at $x = -1$.

(11) (9 points) Find the following indefinite integrals.

(a) $\displaystyle\int \left(2x^3 + \frac{5}{x} + \frac{1}{x^5} - \pi\right) dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{x^4+5}}\, dx$

(c) $\displaystyle\int \sin^2(\theta)\cos(\theta)\, d\theta$

(12) (6 points) Compute the following definite integrals.

(a) $\displaystyle\int_{-1}^{1} \left(6x^5 - 12x^3\right)\, dx$

(b) $\displaystyle\int_{0}^{1} 3x^2(x^3 - 1)^4\, dx$

(13) (10 points) A tank with a square base is to be constructed to hold 10,000 cubic feet of water. The metal top costs \$6 per square foot, and the concrete sides and bottom cost \$4 per square foot. What dimensions x and y yield the lowest cost of materials?