



Name: _____

This is a closed-notes, closed book exam. No calculators, no computers, etc. Put phones away. Answer the questions in the space provided. Put your final answer in a box when appropriate. Each numbered problem is 4 points.

1. Find $\lim_{x \rightarrow 0} \frac{x + \sin(x)}{x}$

2. Find $\lim_{x \rightarrow 2^+} \frac{|2 - x|}{2 - x}$

3. Find $\lim_{x \rightarrow \infty} \frac{2e^x + 3}{x + e^x}$

4. Find $\lim_{x \rightarrow \pi^-} (x - \pi) \tan(x/2)$

5. Find the derivative: $f(x) = x^2 + \frac{1}{x^2} + \pi^2$

6. Find the derivative: $y = \tan(x^2 - 2x - 7)$

7. Find the derivative: $f(x) = \left(\frac{\cos(x)}{1 + x^5}\right)^5$

8. Find the derivative: $y = x^2 \ln(3 + x + x^2)$

9. Find the derivative: $y = \int_3^{x^2+1} \sqrt{1 + \sqrt{t}} dt$

10. Find the indefinite integral: $\int \left(3x^2 + \frac{1}{x^2} + 3 \right) dx$

11. Find the indefinite integral: $\int (x^5 + 5x + 11)^{11} (x^4 + 1) dx$

12. Find the definite integral: $\int_1^2 (x^2 + 2x + 1) dx$

13. Find the definite integral: $\int_{-1}^0 \frac{x}{1+x^2} dx$

14. Use the limit definition of a derivative to compute the derivative of $f(x) = \frac{1}{x}$.

15. Find the global maximum and minimum values of the function $f(x) = \sin(x) + \cos(x)$ on the interval $[0, \pi]$.

The problems on this page concern the function $f(x) = x + \frac{1}{x}$.

16. Find the critical points of $f(x)$.

17. Find the intervals on which $f(x)$ is increasing/decreasing.

18. Identify the locations of any local extrema of $f(x)$

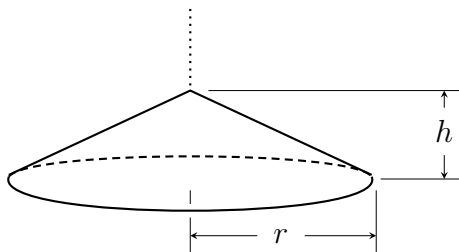
19. Find the intervals on which $f(x)$ is concave up/down.

20. Given the equation $x^2 + xy^2 = y$, use implicit differentiation to find y' .

21. Find the area of the region contained under the graph of $f(x) = \sqrt{x} + 1$, between $x = 1$ and $x = 4$.

22. A ball, tossed straight up, has a constant acceleration of -32 feet per second per second. At time $t = 0$ its velocity is $v(0) = 10$ feet per second, and its position is $s(0) = 6$ feet. Find the position function $s(t)$.

23. Sand falls at a rate of 4 cubic feet per minute, making a conical pile whose height h is always half its radius r . Find the rate of change of the radius r (in feet/min) when $r = 2$ feet.



Geometry formula: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

24. Find $\int_0^3 \sqrt{9-x^2} dx$.

(Hint: consider this as area under a graph.)

25. A 10-foot-high shed with flat roof and three walls is to be constructed, as illustrated below. It is required that the shed have 800 square feet of floor space. Materials for the roof cost \$2 per square foot. Materials for the walls cost \$1 per square foot. Materials for the floor cost \$3 per square foot. What dimensions x and y will minimize the total cost of materials?

