MATH 200 – Final Exam 
VCU Sections 1,2,4,5
December 12, 2012

Name: ____________________________

Directions. Answer the questions in the space provided. This is a closed-notes, closed book exam; no calculators, no computers and no formula sheets. Each problem (or part thereof) is 2 points. You have three hours.

1. (Warmup) Fill in each entry in the middle column with one o the symbols $<$, $>$, or $=$ to express the relationship between the quantities to either side. Rows may be solved quickly using an understanding of course topics, or more slowly by brute force computation.

Left column	$>$ $=$ $<$	Right column
$\sin \pi/4$		$\tan \pi/4$
π		e
$\int_{-1}^{1} x^3 dx$		$\int_{0}^{1} x^3 dx$
$\int_{-1}^{1} x^2 dx$		$\int_{0}^{1} x^2 dx$
$f'(1)$ where $f(x) = x^e$		$g'(1)$ where $g(x) = e^x$
$f'(1,000)$ where $f(x) = x^e$		$g'(1,000)$ where $g(x) = e^x$
number of vertical asymptotes of $f(x) = \dfrac{x^2+2x+1}{x^2+4x+3}$		number of vertical asymptotes of $g(x) = \dfrac{x+1}{x+3}$
number of vertical asymptotes of $y = \ln(x)$		number of horizontal asymptotes of $y = e^x$
number of points on the graph of $x^2 + y^2 = 4$ at which the tangent line has slope $m = 0$		number of points on the graph of $x^2 + y^2 = 4$ at which the tangent line to has slope $m = -1$
$\lim_{x\to\infty} \dfrac{x^4}{x^{44}}$		$\lim_{x\to-\infty} \dfrac{1}{x}$
$\sum_{k=1}^{33} 23$		$\sum_{k=3}^{35} 23$
$\sum_{k=1}^{33} 23k$		$\sum_{k=3}^{35} 23k$
$\tan^{-1}(1)$		$\lim_{x\to\infty} \tan^{-1}(x)$
$f''(0)$ where $f(x) = x^2$		$g''(0)$ where $g(x) = x^3$
number of inflection points for $f(x) = x^3$		number of inflection points for of $f(x) = \sin(x)$

2. Warm up. (Quick answer.)

(a) $\lim_{x\to 0} \frac{\sin(x)}{x} =$

(b) $\lim_{x\to \infty} \frac{\sin(x)}{x} =$

(c) $\lim_{x\to \pi/2} \frac{\sin(x)}{x} =$

(d) $\frac{d}{dx}\left[\frac{\sin(x)}{x}\right] =$

(e) $\sin\left(\frac{4\pi}{3}\right) =$

(f) $\frac{d}{dx}\left[\int_1^x \frac{\sin(t)}{t}\,dt\right] =$

(g) $\lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h} =$

(h) $\frac{d}{dx}\left[x^3 \sin(x)\right] =$

(i) $\int \frac{1}{\sqrt{1-x^2}} dx =$

(j) $\frac{d}{dx}\left[\sin^{-1}(x)\right] =$

(k) $\sin(\tan^{-1} x) =$

(l) $\tan^{-1}(\sqrt{3}) =$

(m) $\lim_{x \to \infty} \tan^{-1}(x) =$

(n) $\ln\left(\frac{1}{\sqrt{e}}\right) =$

(o) $\lim_{x \to 0^+} \ln(x) =$

(p) $\lim_{x \to 1} \ln(x) =$

(q) $\int_1^e \frac{1}{x}\, dx =$

(r) If $f(x) = e^{x+3}$, then $f^{-1}(x) =$

(s) $\frac{d}{dx}\left[\sqrt[5]{x^7}\right] =$

(t) $\int_0^3 x^2\, dx =$

3. Evaluate each limit, or explain why it does not exist.

(a) $\lim_{h \to 0} \dfrac{\dfrac{1}{2+h} - \dfrac{1}{2}}{h}$

(b) $\lim_{x \to -5} \dfrac{x^2 + 3x - 10}{x + 5}$

4. Use the limit definition of the derivative to find the derivative of the function $f(x) = 3x^2$.

5. Let the function f be defined as follows. $f(x) = \begin{cases} x + c & x < 1 \\ x^2 + c^2 & 1 \leq x \end{cases}$

For what values of c, if any, is f continuous at $x = 1$?

6. Find the indicated derivatives.

(a) $\dfrac{d}{dx}\left[\sqrt{x^3+x^2+1}\,\right] =$

(b) $\dfrac{d}{dx}\left[\,x\ln(\sec(3x))\,\right] =$

7. The equation $x^3 + y^3 + 2xy = 4$ determines a curve in the xy-plane. Find the slope of the tangent line to the curve at the point $(1, 1)$.

8. Find the following indefinite integrals.

(a) $\displaystyle\int \frac{\cos(x)}{2\sin(x)+1}\,dx =$

(b) $\displaystyle\int \left(x^3 - 4x + 2\right)^3 \left(3x^2 - 4\right)\,dx =$

9. Find the following definite integrals.

(a) $\displaystyle\int_{-1}^{1} x\sqrt{x^2+3}\,dx =$

(b) $\displaystyle\int_{0}^{\sqrt{3}} \frac{1}{1+x^2}\,dx =$

10. Show that the equation $x + 2\cos x = 0$ has at least one real solution.
Explain which major theorem in Calculus you used to answer this question and how it helped you solve the problem.

11. A particle is traveling along the curve $y^2 - x^3 = 1$.
As it reaches the point $(2, 3)$, the y-coordinate is increasing at the rate of 4 cm/s.
How fast is the x-coordinate changing at that instant?

12. Find the area enclosed between the graphs of $y = x^2$ and $y = x^3$.

13. You have a 200 feet of chain link fence to enclose three rectangular pens, as illustrated. What dimensions (i.e. x feet by y feet) yield the greatest total enclosed area?