Name: Richard

TEST 3

MATH 200, SECTION 1

April 23, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (7 points each) Find the indefinite integrals.

(a) $\int \left(x^3 + \frac{1}{x} + e^x\right)\, dx = \boxed{\frac{x^4}{4} + \ln|x| + e^x + C}$

(b) $\int \left(\frac{3}{x^5} + 1\right)\, dx = \int (3x^{-5} + 1)\, dx = 3 \cdot \frac{1}{-5+1} x^{-5+1} + x + C$

$= \boxed{-\frac{3}{4x^4} + x + C}$

(c) $\int (\sec(x)\tan(x) + 3\sin(x))\, dx = \boxed{\sec(x) - 3\cos(x) + C}$

(d) $\int \frac{1}{\sqrt{x}}\, dx = \int x^{-\frac{1}{2}}\, dx = \frac{1}{-\frac{1}{2}+1} x^{-\frac{1}{2}+1} + C = \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C = \boxed{2\sqrt{x} + C}$

(e) $\int \frac{5}{\sqrt{1-x^2}}\, dx = 5 \int \frac{1}{\sqrt{1-x^2}}\, dx = \boxed{5\sin^{-1}(x) + C}$

(f) $\int \frac{x^2+1}{x}\, dx = \int \left(\frac{x^2}{x} + \frac{1}{x}\right) dx = \int \left(x + \frac{1}{x}\right) dx$

$= \boxed{\frac{x^2}{2} + \ln|x| + C}$

2. (8 points) Is the equation $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\, dx = \cos\left(\frac{1}{x}\right) + C$ true or false? Explain.

$\frac{d}{dx}$?

Let's check: $\frac{d}{dx}\left[\cos\left(\frac{1}{x}\right) + c\right] = -\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = \frac{\sin(1/x)}{x^2}$

We got the integrand, so <u>YES</u> it's <u>TRUE</u>.

3. (8 points) Suppose $f(x)$ is a function for which $f'(x) = 2x + \cos(x)$ and $f(\pi) = 0$. Find $f(x)$.

$$f(x) = \int (2x + \cos(x))\,dx = x^2 + \sin(x) + C$$

So $f(x) = x^2 + \sin(x) + C$, but need to find C.

Know $0 = f(\pi) = \pi^2 + \sin(\pi) + C$

$$0 = \pi^2 + 0 + C$$

$$C = -\pi^2$$

Therefore $\boxed{f(x) = x^2 + \sin(x) - \pi^2}$

4. (8 points each) Find the limits.

(a) $\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -x$

$= \boxed{0}$

form $0\cdot\infty$

form $\frac{\infty}{\infty}$

(b) $\lim_{x\to\pi} \frac{\cos(x)+1}{(x-\pi)^2} = \lim_{x\to\pi} \frac{-\sin(x)}{2(x-\pi)\cdot 1} = \lim_{x\to\pi} \frac{-\sin(x)}{2x-2\pi}$

form $\frac{0}{0}$

form $\frac{0}{0}$ again

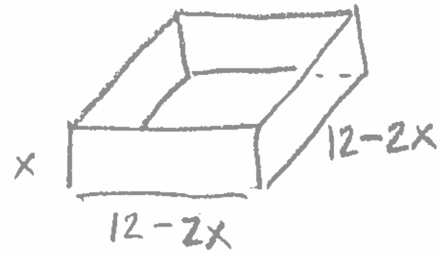
$= \lim_{x\to\pi} \frac{-\cos(x)}{2}$

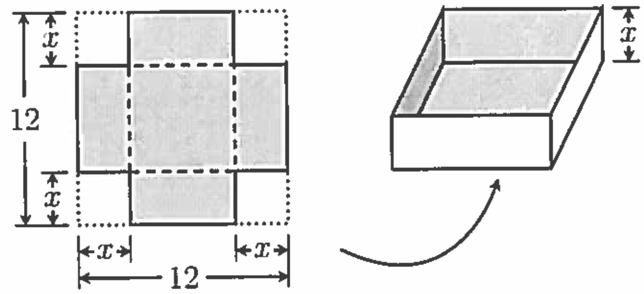
$= \frac{-\cos(\pi)}{2} = \boxed{\frac{1}{2}}$

(c) $\lim_{x\to\infty} \Big(\ln(x+1) - \ln(2x)\Big)$

$= \lim_{x\to\infty} \ln\left(\frac{x+1}{2x}\right) = \ln\left(\lim_{x\to\infty} \frac{x+1}{2x}\right) = \boxed{\ln\left(\frac{1}{2}\right)}$

5. (10 points) An open-top box is made from a 12 by 12 inch piece of cardboard by cutting a square from each corner, and folding up. What should x be to maximize the volume of the box?





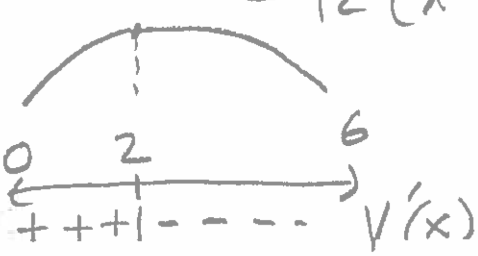
Box has dimensions x by $12-2x$ by $12-2x$, so

$$\text{Volume} = V(x) = x(12-2x)(12-2x)$$
$$V(x) = x(114-48x+4x^2)$$
$$V(x) = 114x - 48x^2 + 4x^3$$

We need to find x giving global maximum of this on $(0, 6)$ ← (Note: x can't exceed $\frac{12}{2} = 6$)

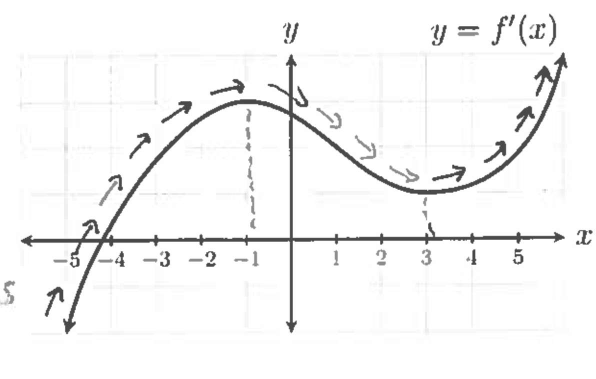
$$V'(x) = 114 - 96x + 12x^2$$
$$= 12(12 - 8x + x^2)$$
$$= 12(x^2 - 8x + 12)$$
$$= 12(x-6)(x-2) = 0$$

Critical points are $x = 2$ and $x = 6$, but only $x = 2$ is in $(0, 6)$



$\boxed{\text{Answer: Volume maximized if } x = 2}$

6. (8 points) Below is the graph of the **derivative** $f'(x)$ of a function $f(x)$. Answer the following question about the function $f(x)$.



(a) On what intervals is $f(x)$ is concave up?

$(-\infty, -1)$ and $(3, \infty)$ because f' increases there, so $f''(x) > 0$.

(b) On what intervals is $f(x)$ is concave down?

$(-1, 3)$ because f' decreases there so $f''(x) < 0$.