Name: _____   April 23, 2021

Directions: Closed book, closed notes, no calculators.   Put all phones, etc., away.   You will need only a pencil or pen.

1. (7 points each) Find the indefinite integrals.

(a) $\displaystyle \int \left(x^3 + \frac{1}{x} + e^x \right) dx$

(b) $\displaystyle \int \left(\frac{3}{x^5} + 1 \right) dx$

(c) $\displaystyle \int \left(\sec(x)\tan(x) + 3\sin(x) \right) dx$

(d) $\displaystyle \int \frac{1}{\sqrt{x}} \, dx$

(e) $\displaystyle \int \frac{5}{\sqrt{1-x^2}} \, dx$

(f) $\displaystyle \int \frac{x^2+1}{x} \, dx$

2. (8 points) Is the equation $\displaystyle \int \frac{\sin\left(\frac{1}{x}\right)}{x^2} \, dx = \cos\left(\frac{1}{x}\right) + C$ true or false? Explain.

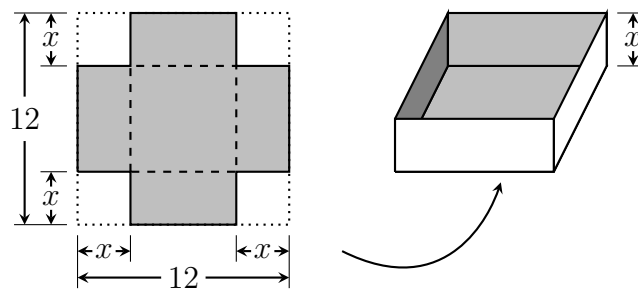3. (8 points) Suppose $f(x)$ is a function for which $f'(x) = 2x + \cos(x)$ and $f(\pi) = 0$. Find $f(x)$.

4. (8 points each) Find the limits.

(a) $\lim\limits_{x \to 0^+} x \ln(x)$

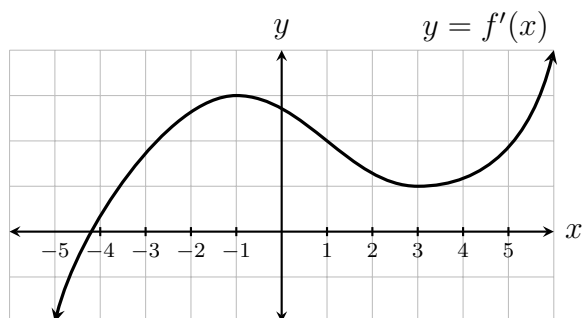(b) $\lim\limits_{x \to \pi} \dfrac{\cos(x) + 1}{(x - \pi)^2}$

(c) $\lim\limits_{x \to \infty} \left(\ln(x + 1) - \ln(2x) \right)$

5. (10 points) An open-top box is made from a 12 by 12 inch piece of cardboard by cutting a square from each corner, and folding up. What should x be to maximize the volume of the box?

6. (8 points) Below is the graph of the **derivative** $f'(x)$ of a function $f(x)$.
   Answer the following question about the function $f(x)$.

   (a) On what intervals is $f(x)$ is concave up?

   (b) On what intervals is $f(x)$ is concave down?