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Test 2 — MATH 200, Section 1

April 2, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (36 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $\frac{d}{dx}\left[x^3 \ln(x)\right] =$ $3x^2 \ln(x) + x^3 \frac{1}{x} = \boxed{3x^2\ln(x) + x^2}$

(b) $\frac{d}{dx}\left[\tan^{-1}(x)\right] =$ $\boxed{\frac{1}{1+x^2}}$

(c) $\frac{d}{dx}\left[\left(2+\ln\left(x^5-x^2\right)\right)^4\right] =$ $\boxed{4\left(2+\ln(x^5-x^2)\right)^3 \frac{5x^4-2x}{x^5-x^2}}$

(d) $\frac{d}{dx}\left[x + \frac{\ln(x)}{x}\right] =$ $1 + \frac{\frac{1}{x}x - \ln(x)\cdot 1}{x^2}$ $\boxed{1+\frac{1-\ln(x)}{x^2}}$

(e) $\frac{d}{dx}\left[\frac{1}{\sqrt{\ln(x)}}\right] =$ $\frac{d}{dx}\left[(\ln(x))^{-\frac{1}{2}}\right] = \frac{-1}{2}(\ln(x))^{-\frac{3}{2}}\frac{1}{x} = \boxed{\frac{-1}{2x\sqrt{\ln(x)^3}}}$

(f) $\frac{d}{dx}\left[\sin^{-1}\left(x^3+3x\right)\right] =$ $\boxed{\frac{1}{\sqrt{1-(x^3+3x)^2}}(3x^2+3)}$

2. (4 points) Find: $\lim_{h\to 0} \frac{\tan^{-1}(2+h) - \tan^{-1}(2)}{h} =$ $\frac{1}{1+2^2} = \boxed{\frac{1}{5}}$

(For $f(x) = \tan^{-1}(x)$, this limit is $f'(2) = \frac{1}{1+2^2}$)

3. (12 points) Given the equation $\ln|x+y| = xy+1$, find y'.

$$\frac{d}{dx}\left[\ln|x+y|\right] = \frac{d}{dx}\left[xy+1\right]$$

$$\frac{1+y'}{x+y} = 1\cdot y + xy' + 0$$

$$1+y' = (x+y)(y+xy')$$

$$1+y' = xy + x^2y' + y^2 + xyy'$$

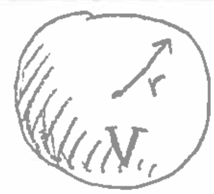
$$1-xy-y^2 = x^2y' + xyy' - y'$$

$$1-xy-y^2 = (x^2+xy-1)\,y'$$

$$\boxed{y' = \frac{1-xy-y^2}{x^2+xy-1}}$$

4. (12 points) A spherical balloon is deflating in such a way that its volume is decreasing at a rate of 18 cubic feet per hour. At what rate is the radius changing when the radius is 3 feet?

Know $\frac{dV}{dt} = -18\ \mathrm{ft}^3/\mathrm{hr}$



V = volume

r = radius

Want $\frac{dr}{dt}$ (when $r=3$)

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\, 3r^2 \frac{dr}{dt}$$

$$-18 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-18}{4\pi r^2} = \frac{-9}{2\pi r^2}$$

Now insert $r=3$:

$$\left.\frac{dr}{dt}\right|_{r=3} = \frac{-9}{2\pi 3^2} = \boxed{\frac{-1}{2\pi}\ \mathrm{ft}/\mathrm{hr}}$$

Sphere formulas Volume: $V = \frac{4}{3}\pi r^3$ Surface area: $S = 4\pi r^2$

5. (12 points) A rocket has a height of $t+t^2$ meters t seconds after it is launched. How high is the rocket when its velocity is 101 meters per second?

position (height) $= S(t) = t + t^2$ meters at time t.

velocity $= V(t) = S'(t) = 1 + 2t$ m/sec

To find when velocity is 101 m/sec, solve

$$V(t) = 101$$
$$1 + 2t = 101$$
$$2t = 100$$
$$t = 50 \text{ sec.}$$

Height at this time is $S(50) = 50 + 50^2 = \boxed{2550 \text{ m.}}$

6. (12 points) Find the locations (x-coordinates) of any local extrema of $f(x) = x^2 e^x$.

$f'(x) = 2xe^x + x^2e^x = e^x x(2+x)$.

This is defined for all x and equals 0 for $x=0$ & $x=-2$.

Thus the $\boxed{\text{critical points are } 0 \text{ and } -2}$

	-2		0		
$+++$		$++++$		$++++$	e^x
$---$		$----$		$++++$	x
$---$		$++++$		$++++$	$(2+x)$
$+++$		$----$		$++++$	$f'(x) = e^x x(2+x)$

f has a local max at $x = -2$

f has a local min at $x = 0$

(by 1st derivative test)

7. (12 points) The graph of the **derivative** $f'(x)$ of a function f is shown below.

(a) State the critical points of f.

$\boxed{x = 5}$ (because $f'(5) = 0$)

(b) State the interval(s) on which f increases.

$\boxed{(-\infty, 5)}$ (that's where $f'(x) > 0$)

(c) State the interval(s) on which f decreases.

$\boxed{(5, \infty)}$ (that's where $f'(x) < 0$)