Name: Richard — **TEST 1** — MATH 200, SECTION 1

March 12, 2021

Directions: Closed book, closed notes, no calculators. Put all phones, etc., away. You will need only a pencil or pen.

1. (10 points) Use a **limit definition** of the derivative to find the derivative of $f(x) = \sqrt{x+1}$.

$$f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x} = \lim_{z\to x} \frac{\sqrt{z+1}-\sqrt{x+1}}{z-x}$$

$$= \lim_{z\to x} \frac{\sqrt{z+1}-\sqrt{x+1}}{z-x} \cdot \frac{\sqrt{z+1}+\sqrt{x+1}}{\sqrt{z+1}+\sqrt{x+1}}$$

$$= \lim_{z\to x} \frac{\sqrt{z+1}^2+\sqrt{z+1}\sqrt{x+1}-\sqrt{x+1}\sqrt{z+1}-\sqrt{x+1}^2}{(z-x)(\sqrt{z+1}+\sqrt{x+1})}$$

$$= \lim_{z\to x} \frac{(z+1)-(x+1)}{(z-x)(\sqrt{z+1}+\sqrt{x+1})}$$

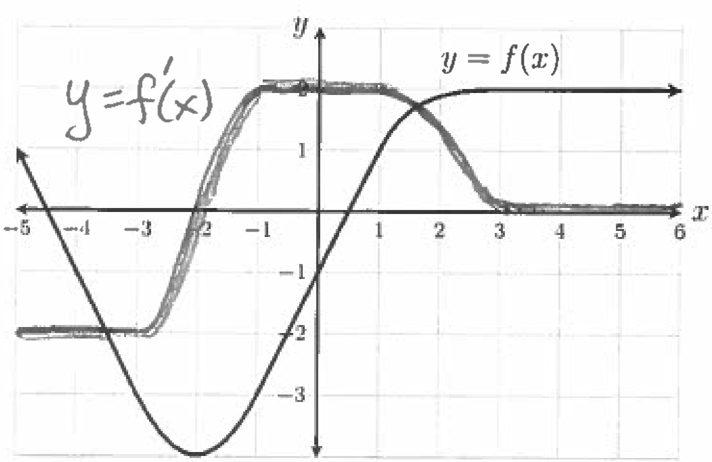
$$= \lim_{z\to x} \frac{\cancel{z-x}}{\cancel{(z-x)}(\sqrt{z+1}+\sqrt{x+1})} = \lim_{z\to x} \frac{1}{\sqrt{z+1}+\sqrt{x+1}} = \boxed{\frac{1}{2\sqrt{x+1}}}$$

2. (10 points) The graph of a function $f(x)$ is sketched below.

(a) Using the same coordinate axis, sketch a graph of the derivative $f'(x)$.

(b) Suppose $g(x) = \dfrac{1}{f(x)}$. Find $g'(0)$.

$$g'(x) = \frac{0\cdot f(x) - 1\cdot f'(x)}{(f(x))^2} = \frac{-f'(x)}{(f(x))^2}$$



Thus $g'(0) = \dfrac{-f'(0)}{(f(0))^2}$

$$= \frac{-2}{(-1)^2} = \boxed{-2}$$

3. (48 points) Find the derivatives of these functions. You do **not** need to simplify your answers.

(a) $f(x) = 5x^7 + 3x - \sqrt{2}$

$$\boxed{f'(x) = 35x^6 + 3}$$

(b) $f(x) = \sin(x) + \sec(x)$

$$\boxed{f'(x) = \cos(x) + \sec(x)\tan(x)}$$

(c) $f(x) = \sin(x)\sec(x)$

$$\boxed{f'(x) = \cos(x)\sec(x) + \sin(x)\sec(x)\tan(x)}$$

(d) $f(x) = \sin\big(\sec(x)\big)$

$$\boxed{f'(x) = \cos\big(\sec(x)\big)\sec(x)\tan(x)}$$

(e) $f(x) = \sec\big(\sin(x)\big)$

$$\boxed{f'(x) = \sec\big(\sin(x)\big)\tan\big(\sin(x)\big)\cos(x)}$$

(f) $f(x) = \dfrac{\tan(x)}{x^2 + e^x}$

$$\boxed{f'(x) = \frac{\sec^2(x)(x^2+e^x) - \tan(x)(2x+e^x)}{(x^2+e^x)^2}}$$

(g) $f(x) = \sqrt{e^x + x}$

$$= (e^x+x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(e^x+x)^{\frac{1}{2}-1}(e^x+1)$$

$$= \boxed{\frac{e^x+1}{2\sqrt{e^x+x}}}$$

(h) $y = \cos\left(e^{x^2+x}\right)$

$$\boxed{f'(x) = -\sin\left(e^{x^2+x}\right)e^{x^2+x}(2x+1)}$$

4. (10 points) Given that $z = w\cos(w)$, find $\frac{d^2z}{dw^2}$.

$$\frac{dz}{dw} = 1 \cdot \cos(w) + w(-\sin(w)) = \cos(w) - w\sin(w)$$

$$\frac{d^2z}{dw^2} = -\sin(w) - 1 \cdot \sin(w) - w\cos(w)$$

$$= \boxed{-2\sin(w) - w\cos(w)}$$

5. (10 points) Find the equation of the tangent line to the graph of $f(x) = e^{-x}$ at $(0, f(0))$.

$f'(x) = e^{-x}(-1) = -e^{-x}$

Therefore the slope of the tangent line is $f'(0) = -e^{-0} = -e^{0} = -1$. A point on the line is $(0, e^{-0}) = (0,1)$. By the point-slope formula the equation of the line is

$y - y_0 = m(x - x_0) \rightsquigarrow y - 1 = -1(x-0) \rightsquigarrow \boxed{y = -x+1}$

6. (10 points) Find all x for which the tangent to the graph of $y = e^x - 2x$ at $(x, f(x))$ is horizontal.

We need to solve the equation

$$f'(x) = 0$$

$$e^x - 2 = 0$$

$$e^x = 2$$

$$\ln(e^x) = \ln(2)$$

$$\boxed{x = \ln(2)}$$