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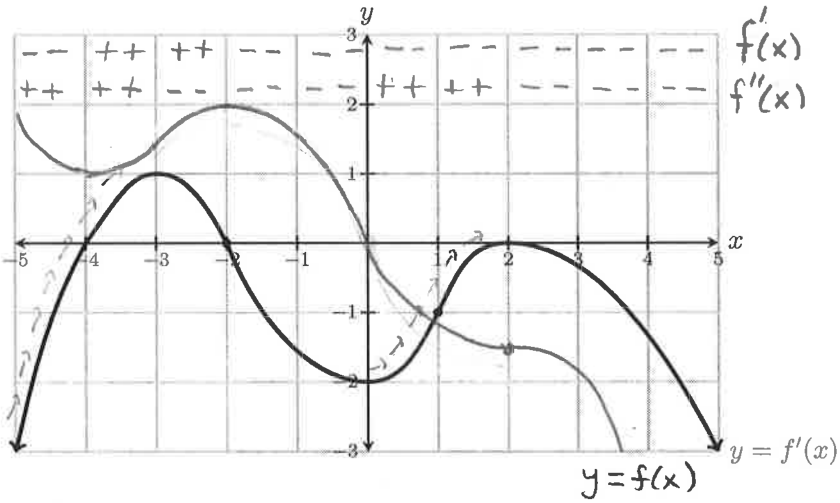
MATH 200 – TEST 3 ☞

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

November 19, 2012

1. (10 pts.) The graph $y = f'(x)$ of **the derivative** of a function $f(x)$ is shown. Answer the questions about $f(x)$.



(a) State the intervals on which the function $f(x)$ increases. $(-4,-2)$ (where $f'(x)>0$)

(b) State the intervals on which the function $f(x)$ decreases. $(-\infty,-4)$, $\underbrace{(-2,\infty)}_{}$ or $(-2,2)\cup(2,\infty)$ (where $f'(x)<0$)

(c) State the intervals on which the function $f(x)$ is concave up. $(-\infty,-3)$, $(0,2)$ (where $f'(x)$ increases)

(d) State the intervals on which the function $f(x)$ is concave down. $(-3,0)$, $(2,\infty)$ (where $f'(x)$ decreases)

(e) Suppose $f(0) = 0$. Using the above information (and coordinate axis), sketch the graph of $f(x)$.

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2. (15 pts.) Find and identify all relative extrema of the function $f(x) = 2-3x^4-8x^3-6x^2$ on the interval $\mathbb{R} = (-\infty,\infty)$. State the extrema in the coordinate form (x, y).

$$f'(x) = -12x^3-24x^2-12x$$
$$= -12x(x^2+2x+1)$$
$$= -12x(x+1)^2$$

0   -1

critical points are 0, -1

	-1		0		
+ + +		+ + +		- - - - - -	$f'(x)$

First Derivative test says

$$\boxed{\begin{array}{l}\text{Local max at}\\ (0, f(0)) = (0,2)\\ \text{No local min}\end{array}}$$

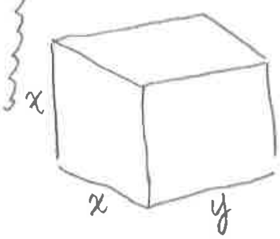
3. (15 pts.) US Postal Service regulations state that the length plus girth of a package cannot exceed 108 inches. You must mail a package whose width and height are equal, and with the greatest possible volume. Find the dimensions of the package.

height = with = x
length = y.
girth = $4x$

length + girth = 108
$y + 4x = 108$
$y = 108-4x$ ← $0 \le x \le 27$

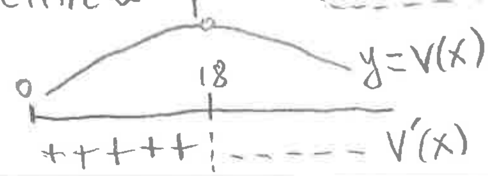
Volume = $xxy = x^2y = x^2(108-4x)$

Volume = $V(x) = 108x^2 - 4x^3$ ← Maximize this on $[0, 27]$



$V'(x) = 216x - 12x^2 = 12x(18-x) = 0$

critical points 0 18



Max volume when $x = 18$

$y = 108 - 4 \cdot 18 = 36$

Answer:
length = 36"
width = height = 18"

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4. (20 points) Evaluate the following limits.

(a) $\lim_{x\to\pi} \frac{1+\cos x}{(\pi - x)^2} = \lim_{x\to\pi} \frac{-\sin x}{2(\pi - x)(-1)} = \lim_{x\to\pi} \frac{-\cos x}{2} = \frac{-\cos\pi}{2} = \frac{-(-1)}{2} = \boxed{\frac{1}{2}}$

form $\frac{0}{0}$ form $\frac{0}{0}$

(b) $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = \boxed{0}$

form $\infty \cdot 0$ form $\frac{\infty}{\infty}$

5. (24 points) Find the indicated indefinite integrals.

(a) $\int \left(7 + 7x + \sqrt[5]{x^2}\right) dx = \int \left(7 + 7x + x^{2/5}\right) dx = 7x + \frac{7}{2}x^2 + \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + C$

$= \boxed{7x + \frac{7}{2}x^2 + \frac{5}{7}x^{7/5} + C}$

(b) $\int \left(e^{4x} + 4\cos x + 20\right) dx = \boxed{\frac{1}{4}e^{4x} + 4\sin x + 20x + C}$

(c) $\int \frac{2x}{x^2}\, dx = \int \frac{2}{x}\, dx = 2\int \frac{1}{x}\, dx = \boxed{2\ln|x| + C}$

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6. (8 pts.) Is the equation $\int (1 + \ln x)\, dx = x + \ln x + C$ true or false? Justify your answer.

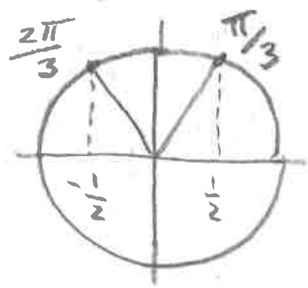
Check: $\frac{d}{dx}\left[x + \ln x + C\right] = 1 + \frac{1}{x} + 0 \neq 1 + \ln x.$

False $x + \ln x + C$ is not an antiderivative of $1 + \ln x$.

7. (8 pts.) Suppose $f(x)$ is a function for which $f'(x) = -\sin(x)$ and $f(2\pi/3) = -3$. Find $f(x)$.

$f(x) = \int -\sin(x)\, dx = \cos(x) + C$

$f(x) = \cos(x) + C$ ← {just need to find C}



$f(2\pi/3) = \cos(2\pi/3) + C$

$-3 = -\frac{1}{2} + C$

$C = -3 + \frac{1}{2} = -\frac{5}{2}$

$\boxed{f(x) = \cos(x) - 5/2}$