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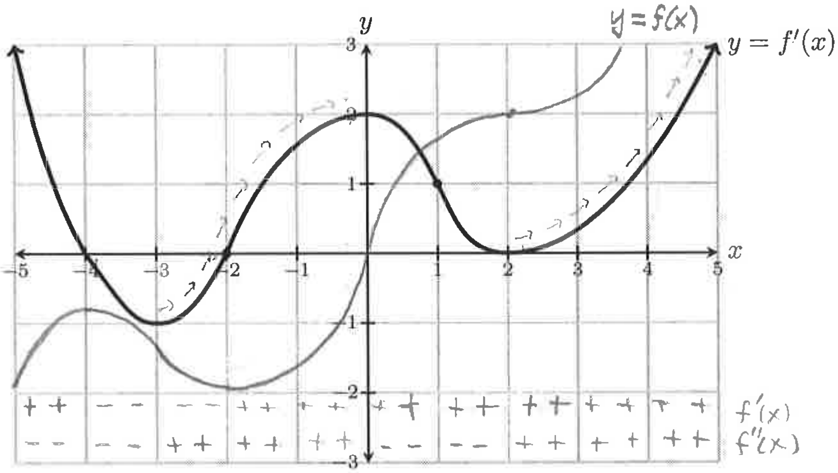
MATH 200 – TEST 3

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

November 19, 2012

1. (10 pts.) The graph $y = f'(x)$ of **the derivative** of a function $f(x)$ is shown. Answer the questions about $f(x)$.



(a) State the intervals on which the function $f(x)$ increases. $(-\infty, -4)$ and $(-2, \infty)$ or $(-2,2)\cup(2,\infty)$ (where $f'(x)>0$)

(b) State the intervals on which the function $f(x)$ decreases. $(-4,-2)$ (where $f'(x)<0$)

(c) State the intervals on which the function $f(x)$ is concave up. $(-3,0)$, $(2,\infty)$ (where $f'(x)$ increases)

(d) State the intervals on which the function $f(x)$ is concave down. $(-\infty,-3)$, $(0,2)$ (where $f'(x)$ decreases)

(e) Suppose $f(0) = 0$. Using the above information (and coordinate axis), sketch the graph of $f(x)$.

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2. (15 pts.) Find and identify all relative extrema of the function $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$ on the interval $\mathbb{R} = (-\infty, \infty)$. State the extrema in coordinate form (x, y).

$$f'(x) = 12x^3 - 24x^2 + 12x$$
$$= 12x(x^2 - 2x + 1)$$
$$= 12x(x-1)^2$$

0   1

critical points are 0, 1

0   1

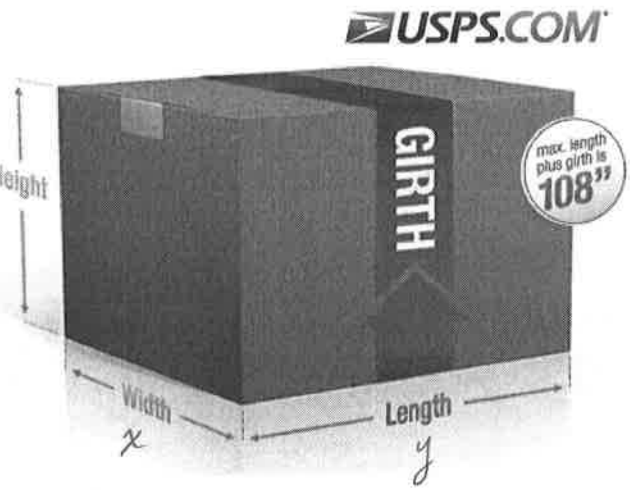
– – – – – | + + + + + $f'(x)$

First Derivative Test says:

Local min at $(0, f(0)) = (0, 1)$.

No local max

3. (15 pts.) US Postal Service regulations state that the length plus girth of a package cannot exceed 108 inches. You must mail a package whose width and height are equal, and with the greatest possible volume. Find the dimensions of the package.



height = width = x } girth = $4x$
length = y

know $4x + y = 108$
$y = 108 - 4x$ ← $0 \le x \le \frac{108}{4} = 27$

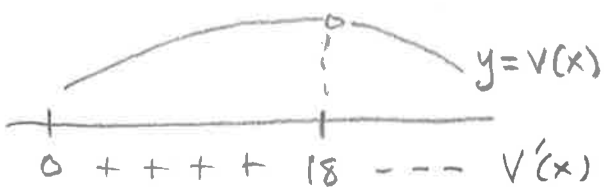
x (height), x (width), y (length)

Volume $= xxy = x^2 y = x^2(108-4x)$

Volume $= V(x) = 108x^2 - 4x^3$ ← maximize this on $[0, 27]$

$V'(x) = 216x - 12x^2$
$= 12x(18-x) = 0$

Critical points: 0, 18



$x = 18$ gives max vol.

$y = 108 - 4 \cdot 18$
$= 108 - 72 = 36$

Answer:
length = 36''
width = height = 18''

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4. (20 points) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to\pi} \frac{(x-\pi)^2}{1+\cos x} = \lim_{x\to\pi} \frac{2(x-\pi)}{-\sin x} = \lim_{x\to\pi} \frac{2}{-\cos x} = \frac{2}{-\cos\pi} = \frac{2}{-(-1)} = \boxed{2}$

form $\frac{0}{0}$ form $\frac{0}{0}$

(b) $\displaystyle\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{-x^2}{x} = \lim_{x\to 0^+} -x$

$= \boxed{0}$

form $0\cdot\infty$ form $\frac{\infty}{\infty}$

5. (24 points) Find the indicated indefinite integrals.

(a) $\int \frac{2x}{x^2}\, dx = \int \frac{2}{x} dx = 2\int \frac{1}{x} dx = \boxed{2 \ln|x| + C}$

(b) $\int \left(x^2 + \sqrt[3]{x^2}\right) dx = \int \left(x^2 + x^{\frac{2}{3}}\right) dx = \frac{x^3}{3} + \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{x^3}{3} + \frac{3}{5} x^{\frac{5}{3}} + C$

$= \boxed{\frac{x^3}{3} + \frac{3}{5}\sqrt[3]{x^5} + C}$

(c) $\int \left(4e^{-x} + \sin x + 3\right) dx = \boxed{-4e^{-x} - \cos x + 3x + C}$

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6. (8 pts.) Is the equation $\int (1 + \ln x)\, dx = x \ln x + C$ true or false? Justify your answer.

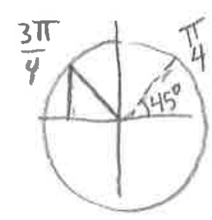
Check $\frac{d}{dx}\left[x \ln x + C\right] = (1)\ln x + x\left(\frac{1}{x}\right) + 0$

$= \ln x + 1$

$= \boxed{1 + \ln x}$ YES checks back TRUE

7. (8 pts.) Suppose $f(x)$ is a function for which $f'(x) = \cos(x)$, and $f(3\pi/4) = 2$. Find $f(x)$.

$f(x) = \int \cos x\, dx = \sin x + C$ ← now we must find C.



$f(3\pi/4) = \sin\left(\frac{3\pi}{4}\right) + C$

$2 = \frac{\sqrt{2}}{2} + C$

$C = 2 - \frac{\sqrt{2}}{2}$

$\boxed{f(x) = \sin(x) + 2 - \frac{\sqrt{2}}{2}}$