VCU

MATH 200

CALCULUS I

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TEST 2

March 20, 2015

Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Score: ⎯⎯⎯⎯⎯⎯⎯⎯⎯

Directions. Answer the questions in the space provided. Unless noted otherwise, you must show and explain your work to receive full credit. Put your final answer in a ⎯box⎯ when appropriate.

This is a closed-book, closed-notes test. Calculators, computers, etc., are not used.

1. **(20 points) Warmup: short answer.**

    (a) $\tan^{-1}(1) =$

    (b) $\cos^{-1}(-1/2) =$

    (c) $e^{1+\cos(\pi)} =$

    (d) $\ln(25) + 2\ln\left(\dfrac{e}{5}\right) =$

    (e) If $f(x) = \ln(x)$, then $f^{-1}(x) =$

    (f) If $f(x) = \ln(x)$, then $f'(x) =$

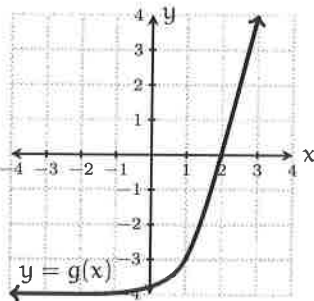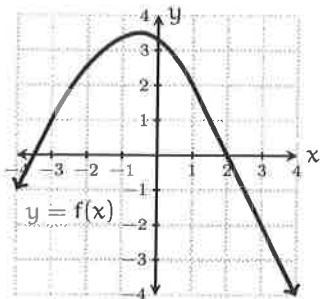   **(g)** $\lim\limits_{h \to 0} \dfrac{\ln(8+h) - \ln(8)}{h} =$

    (h) $\dfrac{d}{dx}\left[\sin(x^{10})\right] =$

   **(i)** $\dfrac{d}{dx}\left[\dfrac{1}{x} + \tan(x)\right] =$

    (j) $\dfrac{d}{dx}\left[\tan^{-1}(x)\right] =$

2. (5 points) Suppose $f(x)$ is the number of gallons of fuel in a rocket when it is x miles above the Earth's surface. Explain, in non-mathematical terms, what the statement $f'(20) = -8$ means.

3. (5 points) Two functions $f(x)$ and $g(x)$ are graphed below. Let $h(x) = f(x)g(x)$. Estimate $h'(1)$. Show your work.

4. (5 points) A function $f(x)$ is graphed below. Using the same coordinate axis, sketch the graph of the derivative $f'(x)$.
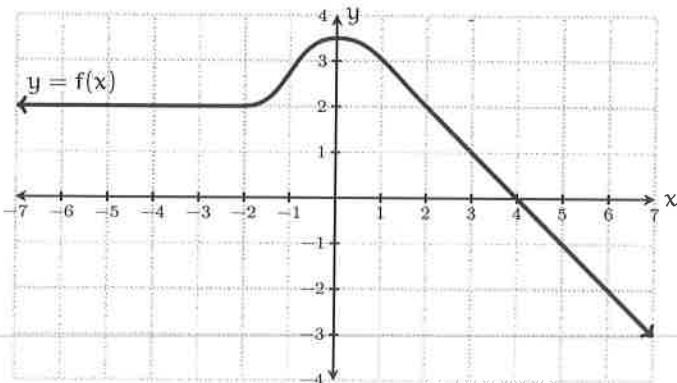
5. (20 points) Find the following derivatives.

(a) $\dfrac{d}{dx}\left[\ln(\tan(x))\right] =$

(b) $\dfrac{d}{dx}\left[\sqrt{\dfrac{x^2+5}{e^x}}\right] =$

(c) $\dfrac{d}{dx}\left[\sqrt[3]{x}\,\sin(x)\right] =$

(d) $\dfrac{d}{dx}\left[e^{1+e^{-x}}\right] =$

6 (10 points) Use logarithmic differentiation to find the derivative of the function $y = (x^2 + 1)^x$.

7 (10 points) Use the limit definition of the derivative to find the derivative of the function $f(x) = x^2 + 1$.

8. (5 points) Simplify: $\tan\left(\cos^{-1}(x)\right) =$

9. (10 points) Suppose $f(x) = \dfrac{5}{x}$.

Find the **equation** of the line tangent to the graph of $y = f(x)$ at the point $(5, f(5))$.

10. (10 points) This question concerns the equation $x^3 + y^3 = 4xy$.

    (a) Use implicit differentiation to find $\dfrac{dy}{dx}$.

    (b) Use your answer from part (a) to find the slope of the tangent line to the graph of $x^3 + y^3 = 4xy$ at the point $(2, 2)$.