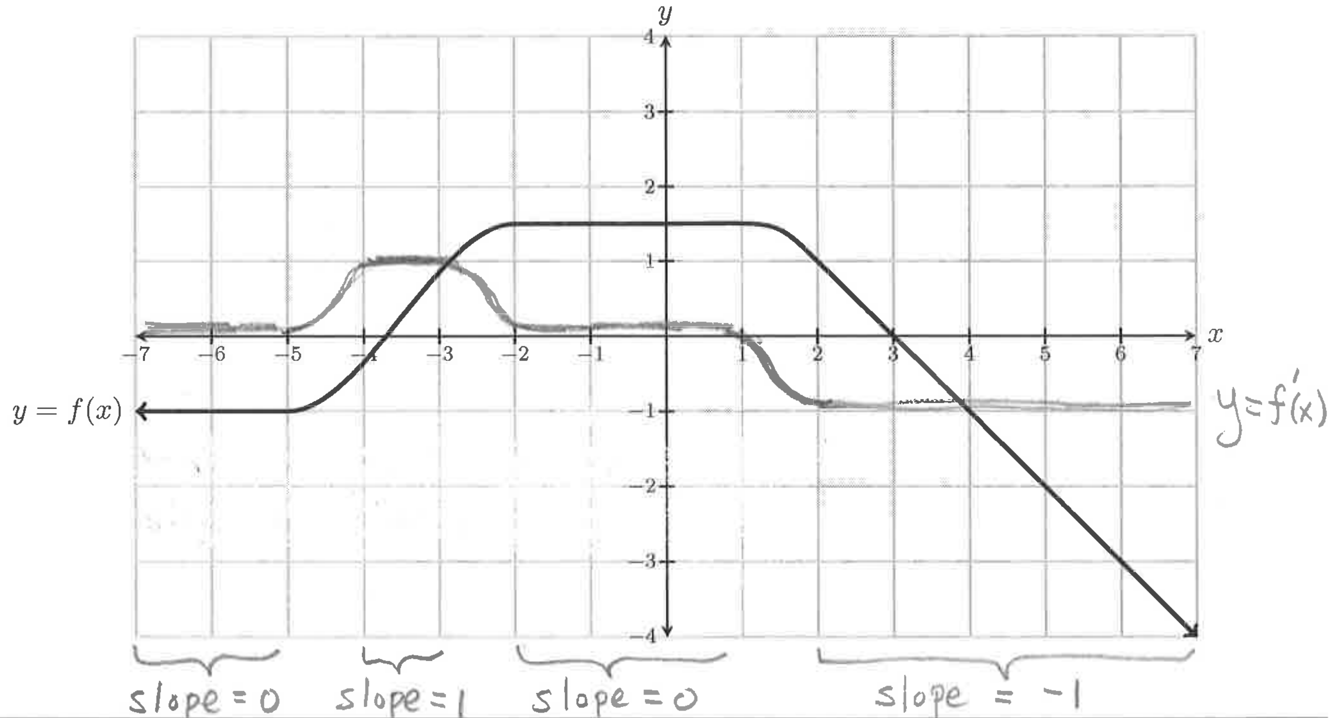
Name: ______________________

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

MATH 200 – TEST 2 ○

October 15, 2012

1. (10 pts.) The graph of a function $f(x)$ is shown. Using the same coordinate axis, sketch the graph of $y = f'(x)$.



2. (10 pts.) Find all points (x, y) on the graph of $y = \frac{1}{x-4} + x - 4$ where the tangent line is horizontal.

$y = (x-4)^{-1} + x - 4$

$$\text{Slope} = \frac{dy}{dx} = -(x-4)^{-2}(1) + 1$$

$$= \frac{-1}{(x-4)^2} + 1$$

We seek x that makes slope $= 0$:

$$0 = \frac{-1}{(x-4)^2} + 1$$

$$\frac{1}{(x-4)^2} = 1$$

$$1 = (x-4)^2$$

$$1 = x^2 - 8x + 16$$

$$0 = x^2 - 8x + 15$$

$$0 = (x-5)(x-3)$$

$x = 5$ $x = 3$ (these x values make slope $= 0$)

Point A

$(5, f(5)) = (5, \frac{1}{5-4} + 5 - 4)$

$= \boxed{(5, 2)}$

Point B

$(3, f(3)) = (3, \frac{1}{3-4} + 3 - 4)$

$= \boxed{(3, -2)}$

Answer $(5,2)$ and $(3,-2)$

3. (14 pts.) Find the indicated derivatives.

(a) $f(\theta) = \sqrt{\theta^5} + e + e^{\pi\theta}$ $= \theta^{5/2} + e + e^{\pi\theta}$

$f'(\theta) = \boxed{\frac{5}{2}\theta^{\frac{3}{2}} + \pi e^{\pi\theta}} = \boxed{\frac{5}{2}\sqrt{\theta^3} + \pi e^{\pi\theta}}$

$f''(\theta) = \boxed{\frac{15}{4}\theta^{1/2} + \pi^2 e^{\pi\theta}} = \boxed{\frac{15}{4}\sqrt{\theta} + \pi^2 e^{\pi\theta}}$

(b) $\frac{d}{dx}\left[\cos^{-1}(\pi x)\right] =$ $\frac{-1}{\sqrt{1-(\pi x)^2}}\pi = \boxed{\frac{-\pi}{\sqrt{1-\pi^2 x^2}}}$

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4. (21 pts.) Find the indicated derivatives.

(a) $\frac{d}{dx}\left[\ln(x^2+1)\sqrt{3x+1}\right] =$ $\frac{2x}{x^2+1}\sqrt{3x+1} + \ln(x^2+1)\frac{1}{2}(3x+1)^{\frac{1}{2}-1}\cdot 3$

$= \boxed{\frac{2x\sqrt{3x+1}}{x^2+1} + \frac{3\ln(x^2+1)}{2\sqrt{3x+1}}}$

(b) $\frac{d}{dx}\left[\sec(\ln(x^3))\right] =$ $\sec(\ln(x^3))\tan(\ln(x^3))\frac{d}{dx}\left[\ln(x^3)\right]$

$= \sec(\ln(x^3))\tan(\ln(x^3))\frac{3x^2}{x^3}$

$= \boxed{\sec(\ln(x^3))\tan(\ln(x^3))\,{}^{3}/_{x}}$

(c) $\frac{d}{dx}\left[\frac{x^3+x^2+1}{x}\right] =$ $\frac{(3x^2+2x)x - (x^3+x^2+1)(1)}{x^2}$

$= \frac{3x^3+2x^2-x^3-x^2-1}{x^2}$

$= \boxed{\frac{2x^3+x^2-1}{x^2}} = \boxed{2x+1-\frac{1}{x^2}}$

5. (10 pts.) Consider the equation $\sin(xy^3) = y$. Use implicit differentiation to find $\frac{dy}{dx}$.

Take $\frac{d}{dx}$ of both sides, then solve for y'.

$$\frac{d}{dx}[\sin(xy^3)] = \frac{d}{dx}[y]$$

$$\cos(xy^3) \cdot \frac{d}{dx}[xy^3] = y'$$ chain rule

$$\cos(xy^3) \cdot ((1) \cdot y^3 + x \cdot (3y^2 \cdot y')) = y'$$ product rule

$$y^3\cos(xy^3) + 3xy^2y'\cos(xy^3) = y'$$ distribute

$$y^3\cos(xy^3) = y' - 3xy^2y'\cos(xy^3)$$ gather y' terms

$$y^3\cos(xy^3) = y'(1 - 3xy^2\cos(xy^3))$$ pull out a y'

$$\boxed{y' = \frac{y^3\cos(xy^3)}{1 - 3xy^2\cos(xy^3)}}$$ solve for y'

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6. (10 pts.) Use logarithmic differentiation to find the derivative of $f(x) = (\sin(x))^x$.

Let $y = (\sin(x))^x$

Take $\ln$ both sides $\ln y = \ln((\sin x)^x)$

The exponent is unlocked by log property, so

$$\ln y = x \ln(\sin x)$$

Take $\frac{d}{dx}$ of both sides now that we're ready

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[x \ln(\sin x)]$$

Now it's an implicit diff. problem. Take deriv.

$$\frac{y'}{y} = \frac{d}{dx}[x]\ln(\sin x) + x\frac{d}{dx}[\ln(\sin(x))]$$

$$\frac{y'}{y} = \ln(\sin x) + x \cdot \frac{\cos x}{\sin x}$$ so $y' = y(\ln(\sin x) + x\cot x)$

Sub in $y = (\sin x)^x$. $\boxed{y' = ((\sin x)^x)(\ln(\sin x) + x\cot x)}$

7. (10 pts.) This problem concerns a rock that is thrown straight up in the air at time $t = 0$. At time t (in seconds) it has a height of $s(t) = 32t - 16t^2$ feet. Please show your work in answering the following questions.

(a) When does the rock hit the ground?

when $s(t) = \text{height} = 0$ and $t > 0$ so time is moving forward!

$s(t) = 0 = 32t - 16t^2 = -16(t^2 - 2t)$ divide out -16

$0 = t^2 - 2t$

$= t(t-2)$ so $t = 0, 2$ sec so $\boxed{t = 2\text{sec}}$

(b) What is its velocity when it hits the ground?

find $v(t) = s'(t)$ and evaluate at ↑

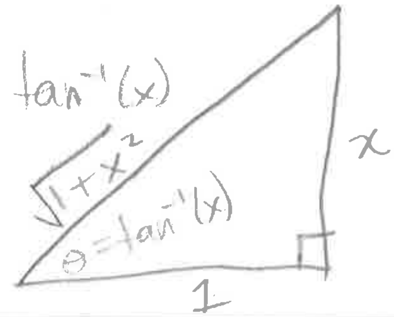
$v(t) = s'(t) = 32 - 32t$

$v(2) = 32 - 32(2) = \boxed{-32 \text{ ft/sec}}$

8. (7 pts.) Simplify: $\sin\left(\tan^{-1}(x)\right) =$ draw triangle for $\tan^{-1}(x)$

$\sin(\tan^{-1}(x))$

$= \sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{x}{\sqrt{1+x^2}}}$



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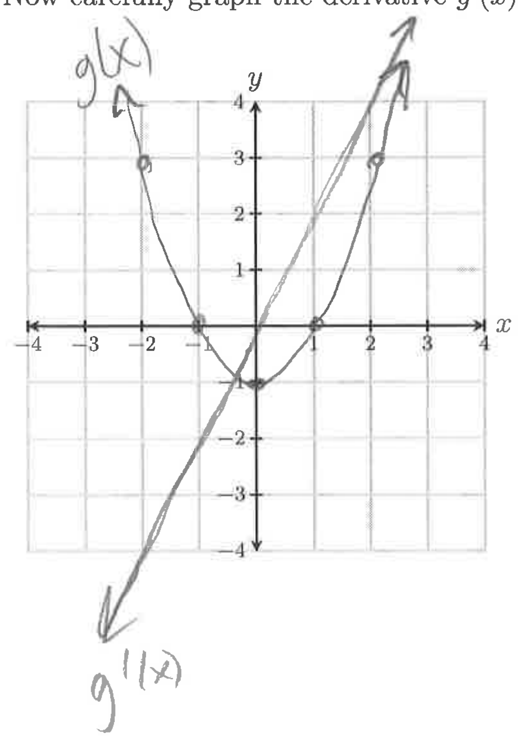
9. (4 pts.)

(a) Graph the function $g(x) = x^2 - 1$ below.

(b) Now carefully graph the derivative $g'(x) = 2x$



10. (4 pts.)

(a) If $f(x) = e^x$, then $f^{-1}(x) = \underline{\ln x}$

(b) Carefully graph $f(x)$ and $f^{-1}(x)$ below.