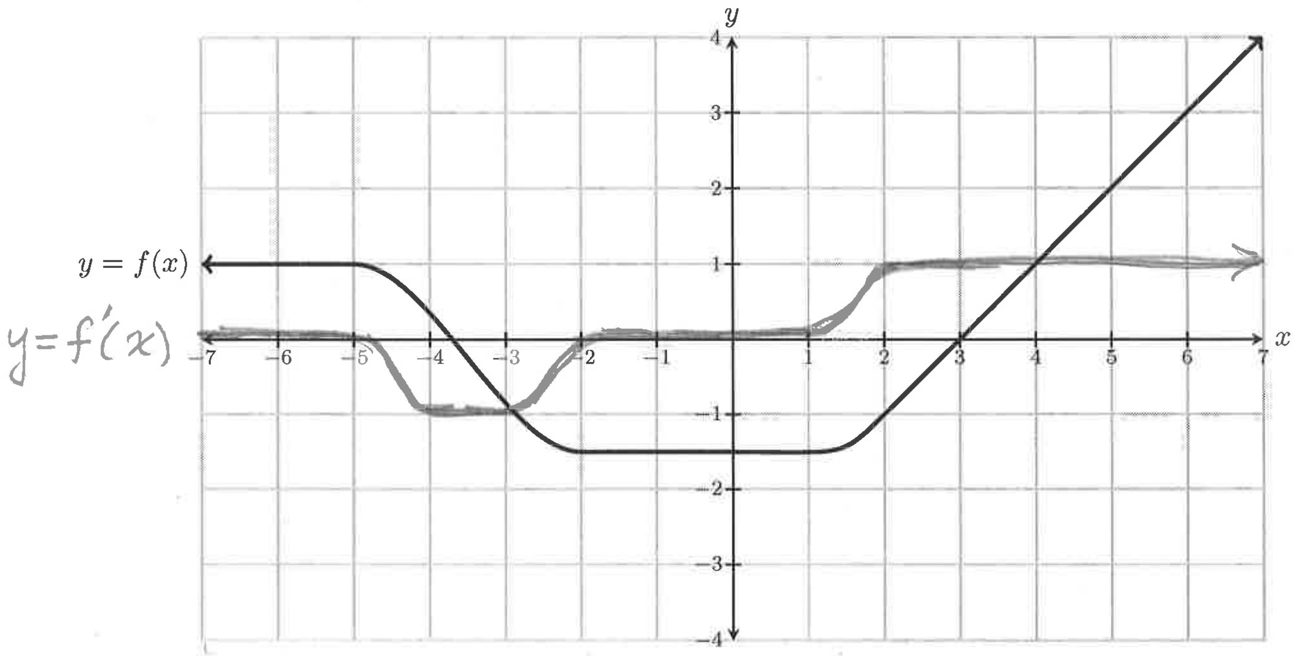
Name: ______________________
I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

MATH 200 – TEST 2 ☽
October 15, 2012

1. (10 pts.) The graph of a function $f(x)$ is shown. Using the same coordinate axis, sketch the graph of $y = f'(x)$.



2. (10 pts.) Find all points (x, y) on the graph of $y = \dfrac{1}{x-5} + x$ where the tangent line is horizontal.

$y = (x-5)^{-1} + x$

Slope $= \dfrac{dy}{dx} = -(x-5)^{-2} + 1 = \dfrac{-1}{(x-5)^2} + 1$

We seek x that makes slope 0.

$\dfrac{-1}{(x-5)^2} + 1 = 0$

$1 = \dfrac{1}{(x-5)^2}$

$(x-5)^2 = 1$

$x^2 - 10x + 25 = 1$

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$

$x = 6$ $\quad$ $x = 4$ (These make slope zero)

Point A:

$(6, f(6)) = \left(6, \frac{1}{6-5} + 6\right)$

$= (6, 7)$

Point B

$(4, f(4)) = \left(4, \frac{1}{4-5} + 4\right)$

$= (4, 3)$

Answer $(6,7)$ and $(4,3)$

3. (14 pts.) Find the indicated derivatives.

(a) $f(\theta) = \pi^2 + \ln(5\theta) + \sqrt{\theta}^9$ $= \pi^2 + \ln(5\theta) + \theta^{9/2}$

$f'(\theta) =$ $0 + \frac{5}{5\theta} + \frac{9}{2}\theta^{7/2} = \frac{1}{\theta} + \frac{9}{2}\theta^{7/2} = \boxed{\frac{1}{\theta} + \frac{9}{2}\sqrt{\theta}^{7}}$

$f''(\theta) =$ $\frac{-1}{\theta^2} + \frac{63}{4}\theta^{5/2} = \boxed{-\frac{1}{\theta^2} + \frac{63}{4}\sqrt{\theta}^{5}}$

(b) $\frac{d}{dx}\left[\frac{x}{x^3 + x^2 + 1}\right] =$ $\frac{(1)(x^3+x^2+1) - x(3x^2+2x)}{(x^3+x^2+1)^2}$

4. (21 pts.) Find the indicated derivatives.

(a) $\frac{d}{dx}\left[e^{3x}\sqrt{x^4+1}\right] =$ $e^{3x}(3)\sqrt{x^4+1} + e^{3x}\frac{1}{2}(x^4+1)^{\frac{1}{2}-1}4x^3$

$= \boxed{3e^{3x}\sqrt{x^4+1} + \frac{2x^3e^{3x}}{\sqrt{x^4+1}}}$

(b) $\frac{d}{dx}\left[\cos^{-1}(\pi x)\right] =$ $\frac{-1}{\sqrt{1-(\pi x)^2}} \cdot \pi = \boxed{\frac{-\pi}{\sqrt{1-\pi^2x^2}}}$

(c) $\frac{d}{dx}\left[\ln(\sec(x^3))\right] =$ $\frac{1}{\sec(x^3)}\frac{d}{dx}\left[\sec(x^3)\right]$

$= \frac{1}{\sec(x^3)}\sec(x^3)\tan(x^3)3x^2$

$= \boxed{3x^2\tan(x^3)}$

5. (10 pts.) Consider the equation $x\cos(y^3) = e^y$. Use implicit differentiation to find $\frac{dy}{dx}$.

Take $\frac{d}{dx}$ of both sides.

$$\frac{d}{dx}[x\cos(y^3)] = \frac{d}{dx}[e^y]$$

$$\frac{d}{dx}[x]\cos(y^3) + x\frac{d}{dx}[\cos(y^3)] = \frac{d}{dx}[e^y]$$

$$(1)\cdot\cos(y^3) + x\cdot(-\sin(y^3)\cdot 3y^2y') = e^y y'$$

$$\cos(y^3) - 3xy^2y'\sin(y^3) = e^y y'$$

Gather terms with y' to right-hand side:

$$\cos(y^3) = e^y y' + 3xy^2y'\sin(y^3)$$ factor out a y'

$$\cos(y^3) = y'(e^y + 3xy^2\sin(y^3))$$ solve for y'

$$\boxed{y' = \frac{\cos(y^3)}{(e^y + 3xy^2\sin(y^3))}}$$

6. (10 pts.) Use logarithmic differentiation to find the derivative of $f(x) = (\cos(x))^x$.

Let $y = (\cos x)^x$. Take ln of both sides:

$$\ln y = \ln((\cos(x))^x)$$

use exponent-unlocking log property: $\ln(x^r) = r\ln(x)$

$$\ln y = x\ln(\cos x)$$

It's an implicit differentiation problem now. $\frac{d}{dx}$ both sides:

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[x\ln(\cos x)]$$ (product rule):

$$\frac{y'}{y} = \frac{d}{dx}[x]\ln(\cos x) + x\frac{d}{dx}[\ln(\cos x)]$$

$$\frac{y'}{y} = \ln(\cos x) + x\cdot\frac{-\sin x}{\cos x}$$

$$y' = y(\ln(\cos x) - x\tan x)$$ Now sub in for $y = (\cos x)^x$

$$\boxed{y' = ((\cos x)^x)(\ln(\cos x) - x\tan x)}$$

7. (10 pts.) This problem concerns a rock that is thrown off a tower at time $t = 0$. At time t (in seconds) it has a height of $s(t) = 48 + 32t - 16t^2$ feet. Please show your work in answering the following questions.

position

(a) When does the rock hit the ground?

when $s(t) = 0$ and $t > 0$

$s(t) = 0 = 48 + 32t - 16t^2$ divide by -16

$t^2 - 2t - 3 = 0$

$(t-3)(t+1) = 0 \Rightarrow t = -1, 3$ sec so when $\boxed{t = 3\text{sec}}$

(b) What is its velocity when it hits the ground?

$= v(3)$ because hits ground @ $t = 3$ sec

$v(t) = s'(t) = 32 - 32t$

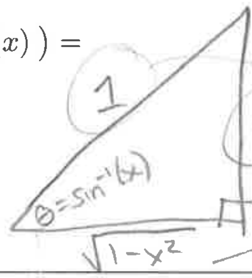
$v(3) = 32 - 32(3) = 32(-2) = \boxed{-64\text{ft/sec}}$

8. (7 pts.) Simplify: $\cos\left(\sin^{-1}(x)\right) =$

$\cos(\sin^{-1}(x))$

$= \cos(\theta) = \dfrac{\text{adj}}{\text{hyp}}$

$= \dfrac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$



← draw triangle for $\sin^{-1}(x)$

because $\sin$ is $\dfrac{\text{opp}}{\text{hyp}} = \dfrac{x}{1}$

by pythag. thm

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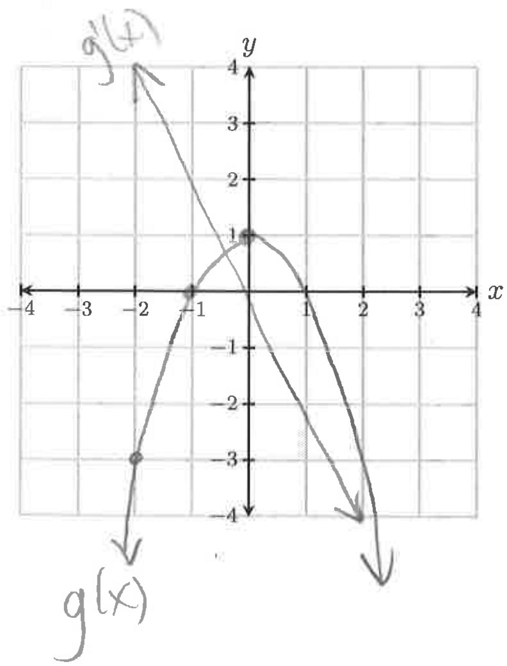
9. (4 pts.)

(a) Graph the function $g(x) = 1 - x^2$ below.

(b) Now carefully graph the derivative $g'(x) = -2x$



(c) (4 pts.)

i. If $f(x) = e^x$, then $f^{-1}(x) = \underline{\ln x}$.

ii. Carefully graph $f(x)$ and $f^{-1}(x)$ below.