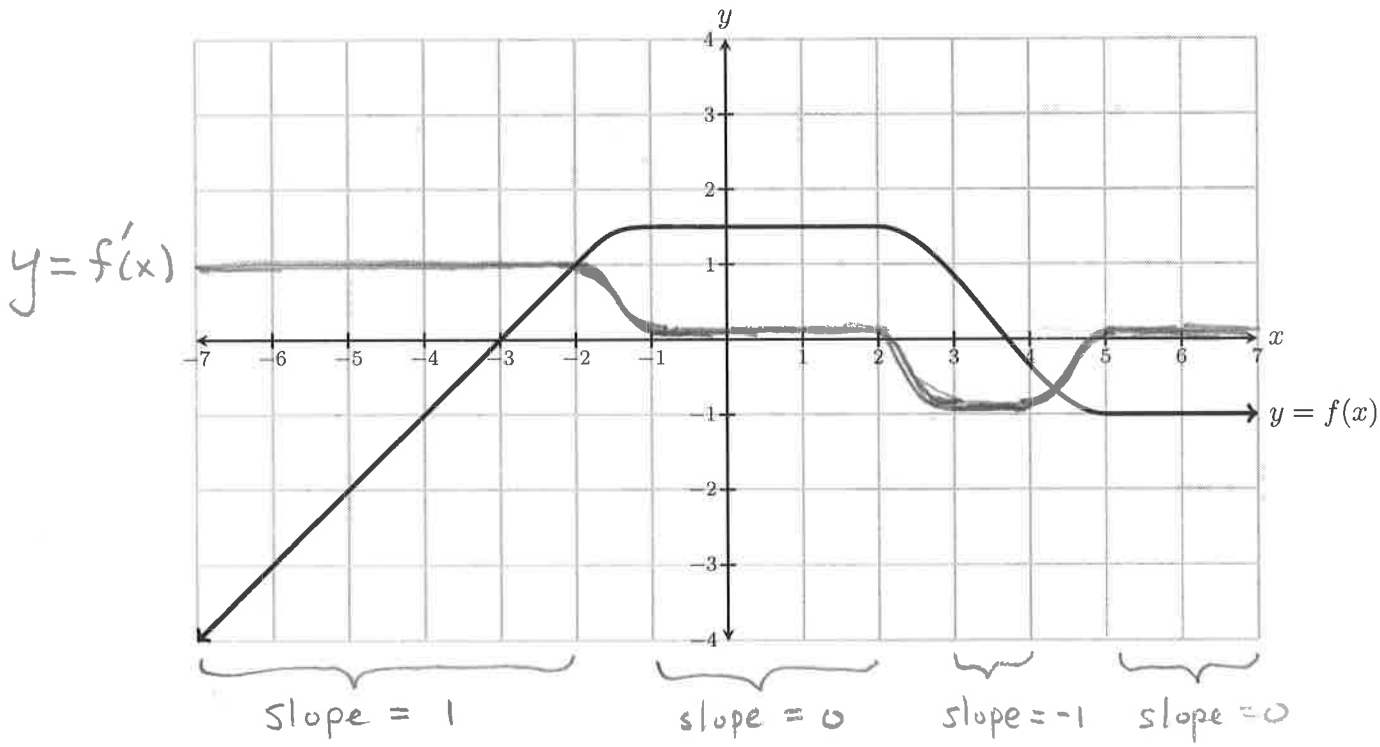
Name: ____________________

I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

MATH 200 – TEST 2 ☾

October 15, 2012

1. (10 pts.) The graph of a function $f(x)$ is shown. Using the same coordinate axis, sketch the graph of $y = f'(x)$.



2. (10 pts.) Find all points (x, y) on the graph of $y = x^2 + \frac{16}{x^2}$ where the tangent line is horizontal.

$y = x^2 + 16x^{-2}$

$$\text{Slope} = \frac{dy}{dx} = 2x - 32x^{-3}$$
$$= 2x - \frac{32}{x^3}$$

We seek x that makes slope 0.

$$2x - \frac{32}{x^3} = 0$$
$$2x^4 - 32 = 0$$
$$x^4 - 16 = 0$$
$$x = \sqrt[4]{16} = \pm 2$$

(these are the x values that make slope = 0)

Point A

$$(2, f(2)) = \left(2, 2^2 + \frac{16}{2^2}\right)$$
$$= (2, 4+4)$$
$$= (2, 8)$$

Point B

$$(-2, f(-2)) = \left(-2, (-2)^2 + \frac{16}{(-2)^2}\right)$$
$$= (-2, 4+4)$$
$$= (-2, 8)$$

$\boxed{\text{Answer } (2,8) \text{ and } (-2,8)}$

3. (14 pts.) Find the indicated derivatives.

(a) $f(\theta) = \sqrt{\theta^5} + \ln(\pi\theta) - \pi^2 = \theta^{\frac{5}{2}} + \ln(\pi\theta) - \pi^2$

$f'(\theta) = \frac{5}{2}\theta^{\frac{3}{2}} + \frac{\pi}{\pi\theta} - 0 = \boxed{\frac{5}{2}\sqrt{\theta}^{\,3} + \frac{1}{\theta}}$

$f''(\theta) = \frac{15}{4}\theta^{\frac{1}{2}} - \frac{1}{\theta^2} = \boxed{\frac{15}{4}\sqrt{\theta} - \frac{1}{\theta^2}}$

(b) $\frac{d}{dx}\left[\,(x^2+x)\sqrt{3x+1}\,\right] = \frac{d}{dx}\left[(x^2+x)(3x+1)^{\frac{1}{2}}\right] =$

$(2x+1)\sqrt{3x+1} + (x^2+x)\frac{1}{2}(3x+1)^{-\frac{1}{2}}\,3 =$

$(2x+1)\sqrt{3x+1} + \frac{3(x^2+x)}{2\sqrt{3x+1}}$

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4. (21 pts.) Find the indicated derivatives.

(a) $\frac{d}{dx}\left[\frac{x^3+x^2+1}{x}\right] = \boxed{\frac{(3x^2+2x)x - (x^3+x^2+1)(1)}{x^2}}$

(b) $\frac{d}{dx}\left[\,(\sec(\ln x))^3\,\right] = 3(\sec(\ln x))^2 \frac{d}{dx}\left[\sec(\ln x)\right]$

$= \boxed{3(\sec(\ln x))^2 \sec(\ln x)\tan(\ln x)\frac{1}{x}}$

(c) $\frac{d}{dx}\left[\,\sec^{-1}(\pi x)\,\right] = \boxed{\frac{\pi}{|\pi x|\sqrt{(\pi x)^2-1}}} = \boxed{\frac{1}{|x|\sqrt{(\pi x)^2-1}}}$

5. (10 pts.) Consider the equation $x\tan(y^3) = y$. Use implicit differentiation to find $\frac{dy}{dx}$.

Take $\frac{d}{dx}$ of both sides:

$$\frac{d}{dx}\left[x\tan(y^3)\right] = \frac{d}{dx}[y]$$

$$\frac{d}{dx}[x]\tan(y^3) + x\frac{d}{dx}\left[\tan(y^3)\right] = y'$$

$$(1)\tan(y^3) + x\left(\sec^2(y^3)\cdot 3y^2\cdot y'\right) = y'$$

Gather terms with y' on right-hand side

$$\tan(y^3) = y' - 3xy^2y'\cdot\sec^2(y^3)$$ and factor out y'

$$\tan(y^3) = y'\left(1 - 3xy^2\sec^2(y^3)\right)$$ and solve for y'

$$\boxed{\frac{\tan(y^3)}{1-3xy^2\sec^2(y^3)} = y'}$$

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6. (10 pts.) Use logarithmic differentiation to find the derivative of $f(x) = x^{\sin(x)}$.

Let $y = x^{\sin x}$. Take $\ln$ of both sides.

$$\ln y = \ln(x^{\sin x})$$

Use log property $\ln(x^r) = r\ln(x)$ for unlocking exponents

$$\ln y = \sin x \ln(x)$$

Now it's an implicit diff. prob. Take $\frac{d}{dx}$ of both sides.

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\sin x \ln x]$$

$$\frac{y'}{y} = \frac{d}{dx}[\sin x]\ln x + \sin x\frac{d}{dx}[\ln x]$$ product rule

$$\frac{y'}{y} = \cos x\ln x + \sin x\cdot\frac{1}{x}$$

$$y' = y\left(\cos x\ln x + \frac{\sin x}{x}\right)$$ plug in $y = x^{\sin x}$

$$\boxed{y' = \left(x^{\sin x}\right)\left(\cos x\ln x + \frac{\sin x}{x}\right)}$$

7. (10 pts.) This problem concerns a rock that is thrown straight up in the air at time $t = 0$. At time t (in seconds) it has a height of $s(t) = 64t - 16t^2$ feet. Please show your work in answering the following questions.

(a) When does the rock hit the ground?

when $s(t) = \text{height} = 0$ and $t > 0$ solve for t

$s(t) = 0 = 64t - 16t^2 = -16(t^2 - 4t)$ divide out -16

$0 = t^2 - 4t = t(t-4)$ so $t = 0, 4$ sec so @ $\boxed{t = 4 \text{ sec}}$

(b) What is its velocity when it hits the ground?

find velocity $v(t)$ and evaluate at

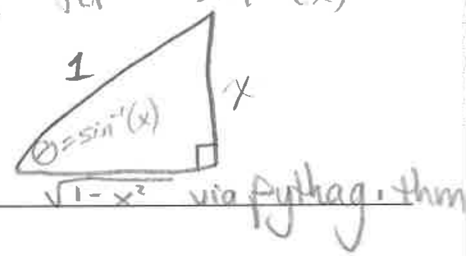
$v(t) = s'(t) = 64 - 32t$

$v(4) = 64 - 32(4) = \boxed{-64 \text{ ft/sec}}$

8. (7 pts.) Simplify: $\tan\left(\sin^{-1}(x)\right) =$ draw the triangle for $\sin^{-1}(x)$

$\tan(\sin^{-1}(x))$

$= \tan(\theta) = \frac{\text{opp}}{\text{adj}} = \boxed{\frac{x}{\sqrt{1-x^2}}}$

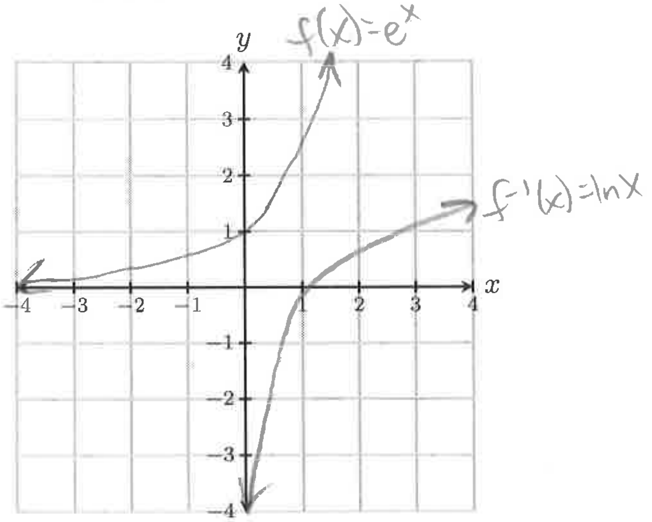


via Pythag. thm

9. (4 pts.)

(a) If $f(x) = e^x$, then $f^{-1}(x) =$ $\ln x$.

(b) Carefully graph $f(x)$ and $f^{-1}(x)$ below.



10. (4 pts.)

(a) Graph the function $g(x) = 1 - x^2$ below.

(b) Now carefully graph the derivative $g'(x) = -2x$