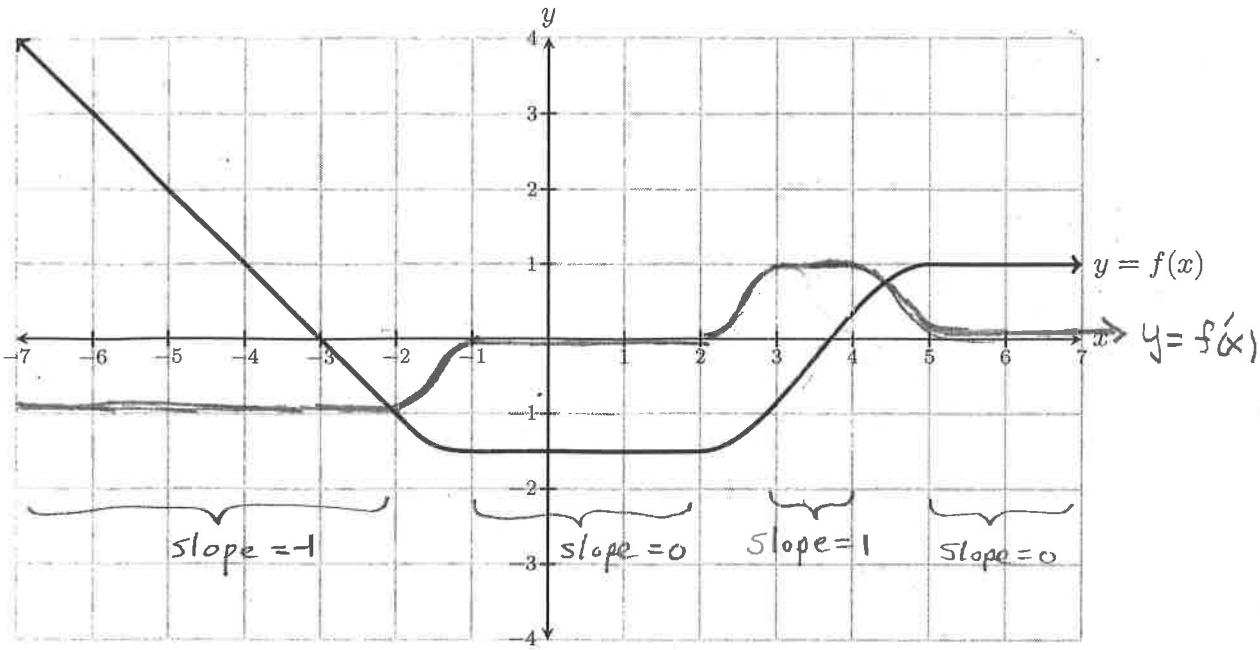
Name: ______________________________
I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

MATH 200 – TEST 2 ●
October 15, 2012

1. (10 pts.) The graph of a function $f(x)$ is shown. Using the same coordinate axis, sketch the graph of $y = f'(x)$.



2. (10 pts.) Find all points (x, y) on the graph of $y = x + \frac{1}{x-3}$ where the tangent line is horizontal.

$$y = x + (x-3)^{-1}$$

$$\text{slope} = \frac{dy}{dx} = 1 - (x-3)^{-2}(1) = 1 - \frac{1}{(x-3)^2}$$

We seek x that makes slope $= 0$.

That is, we must solve $1 - \frac{1}{(x-3)^2} = 0$

$$1 = \frac{1}{(x-3)^2}$$

$$(x-3)^2 = 1$$

$$x^2 - 6x + 9 = 1$$

$$x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$x = 2$ $x = 4$

Point A

$(2, f(2)) = \left(2, 2 + \frac{1}{2-3}\right) = \boxed{(2, 1)}$

Point B

$(4, f(4)) = \left(4, 4 + \frac{1}{4-3}\right) = \boxed{(4, 5)}$

3. (14 pts.) Find the indicated derivatives.

(a) $f(\theta) = 5 + \ln(\pi\theta) + \sqrt{\theta^3}$ $= 5 + \ln(\pi\theta) + \theta^{3/2}$

$f'(\theta) =$ $0 + \dfrac{\pi}{\pi\theta} + \dfrac{3}{2}\theta^{1/2} = \boxed{\dfrac{1}{\theta} + \dfrac{3}{2}\sqrt{\theta}}$

$f''(\theta) =$ $\dfrac{-1}{\theta^2} + \dfrac{3}{4}\theta^{-1/2} = \boxed{\dfrac{-1}{\theta^2} + \dfrac{3}{4\sqrt{\theta}}}$

(b) $\dfrac{d}{dx}\left[\dfrac{x}{x^3+x^2+1}\right] =$ $\boxed{\dfrac{(1)(x^3+x^2+1) - x(3x^2+2x)}{(x^3+x^2+1)^2}}$

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4. (21 pts.) Find the indicated derivatives.

(a) $\dfrac{d}{dx}\left[e^{4x}\sqrt{3x+1}\right] =$ $4e^{4x}\sqrt{3x+1} + e^{4x}\dfrac{1}{2}(3x+1)^{-\frac{1}{2}}(3)$

$= \boxed{4e^{4x}\sqrt{3x+1} + \dfrac{3e^{4x}}{2\sqrt{3x+1}}}$

(b) $\dfrac{d}{dx}\left[\ln\left(\sec(x^3)\right)\right] =$ $\dfrac{1}{\sec(x^3)}\dfrac{d}{dx}\left[\sec(x^3)\right] = \dfrac{1}{\sec(x^3)}\sec(x^3)\tan(x^3)3x^2$

$= \boxed{3x^2\tan(x^3)}$

(c) $\dfrac{d}{dx}\left[\tan^{-1}(\pi x)\right] =$ $\dfrac{1}{1+(\pi x)^2}\cdot\pi = \boxed{\dfrac{\pi}{1+\pi^2x^2}}$

5. (10 pts.) Consider the equation $x\sin(y) = y^3$. Use implicit differentiation to find $\frac{dy}{dx}$.

Take $\frac{d}{dx}$ of both sides.

$$\frac{d}{dx}[x \sin y] = \frac{d}{dx}[y^3]$$

$$\frac{d}{dx}[x]\sin y + x \cdot \frac{d}{dx}[\sin y] = 3y^2 \cdot y'$$ product rule on left-hand side

$$(1) \cdot \sin y + x\cos y \cdot y' = 3y^2 \cdot y'$$

Gather terms with y' on right-hand side

$$\sin y = 3y^2 \cdot y' - x\cos y \cdot y'$$ factor out y':

$$\sin y = y'(3y^2 - x\cos y)$$ solve for y':

$$\boxed{y' = \frac{\sin y}{3y^2 - x\cos y}}$$

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6. (10 pts.) Use logarithmic differentiation to find the derivative of $f(x) = x^{\cos(x)}$.

Let $y = x^{\cos x}$. Take ln of both sides.

$\ln y = \ln(x^{\cos x})$ Use log property to unlock exponent:

$\ln y = \cos x \ln(x)$

Now, it's an implicit problem, take $\frac{d}{dx}$ of both sides:

$$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[\cos x \ln(x)]$$

$$\frac{y'}{y} = \frac{d}{dx}[\cos x]\ln x + \cos x \frac{d}{dx}[\ln(x)]$$ product rule

$$\frac{y'}{y} = -\sin x \ln x + \cos x \cdot \frac{1}{x}$$

$$y' = y\left(-\sin x \ln x + \frac{\cos x}{x}\right)$$ plug in $y = x^{\cos x}$

$$\boxed{y' = (x^{\cos x})\left(-\sin x \ln x + \frac{\cos x}{x}\right)}$$

7. (10 pts.) This problem concerns a rock that is thrown off a tower at time $t = 0$. At time t (in seconds) it has a height of $s(t) = 48 + 32t - 16t^2$ feet. Please show your work in answering the following questions.

(a) When does the rock hit the ground? and when $t > 0$ (forward in time)

when $s(t) = \text{height} = 0$ ft. Now solve for t.

$s(t) = 0 = 48 + 32t - 16t^2 = -16(t^2 - 2t - 3)$, divide out -16

$0 = t^2 - 2t - 3$

$(t-3)(t+1) = 0 \Rightarrow t = 3, -1$ sec so, $\boxed{\text{when } t = 3 \text{ sec}}$

(b) What is its velocity when it hits the ground?

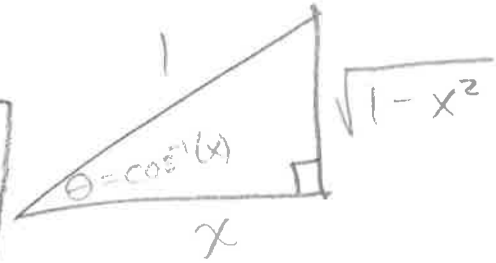
find $v(t)$ and evaluate at

$v(t) = s'(t) = 32 - 32t$

$v(3) = 32 - 32(3) = \boxed{-64 \text{ ft/sec}}$

8. (7 pts.) Simplify: $\sec\left(\cos^{-1}(x)\right) = \dfrac{1}{\cos(\cos^{-1}(x))}$

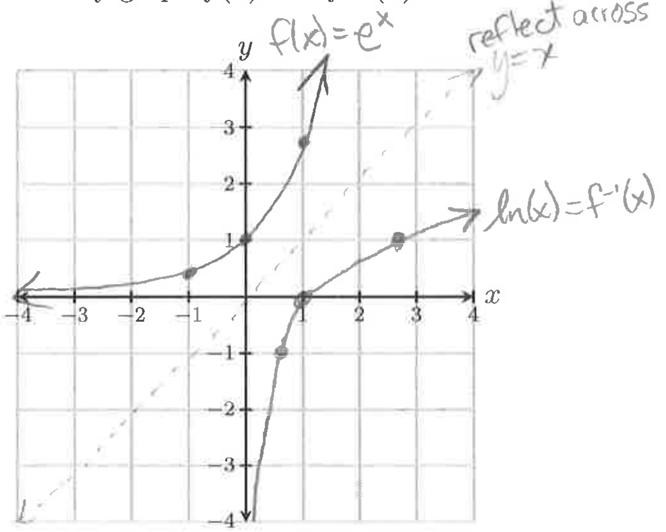
$\dfrac{1}{\cos(\cos^{-1}(x))} = \dfrac{1}{\cos(\theta)} = \dfrac{1}{\frac{adj}{hyp}} = \dfrac{1}{\frac{x}{1}} = \boxed{\dfrac{1}{x}}$



9. (4 pts.)

(a) If $f(x) = e^x$, then $f^{-1}(x) = \underline{\ln x}$.

(b) Carefully graph $f(x)$ and $f^{-1}(x)$ below.



10. (4 pts.)

(a) Graph the function $g(x) = x^2 - 1$ below.

(b) Now carefully graph the derivative $g'(x) = 2x$