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I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

Sept. 14, 2012

MATH 200 – TEST 1 ♡

1. (10 pts.) This problem concerns the functions $f(x) = \dfrac{\sqrt{x-2}}{3+\cos(x)}$ and $g(x) = \sqrt{x} - 2$.

   (a) State the domain of $f(x)$. The denominator causes no problem, since it's always positive (never 0). However, we must have $x - 2 \geq 0$ under the radical. Thus $x \geq 2$. $\boxed{\text{Domain: } [2, \infty)}$

   (b) $f \circ g(x) = f(g(x)) = \dfrac{\sqrt{g(x)-2}}{3+\cos(g(x))} = \dfrac{\sqrt{\sqrt{x}-2-2}}{3+\cos(\sqrt{x}-2)} = \dfrac{\sqrt{\sqrt{x}-4}}{3+\cos(\sqrt{x}-3)}$

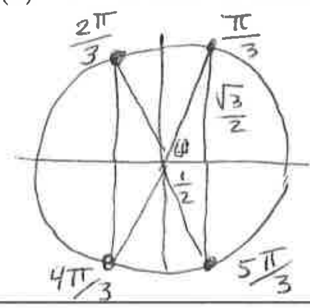
2. (10 pts.) Consider the equation $\frac{1}{3}\tan^2(x) - 1 = 0$. Find all solutions x that lie in the interval $[0, 2\pi)$.

$\frac{1}{3}\tan^2(x) = 1$

$\tan^2(x) = 3$

$\tan(x) = \pm\sqrt{3}$

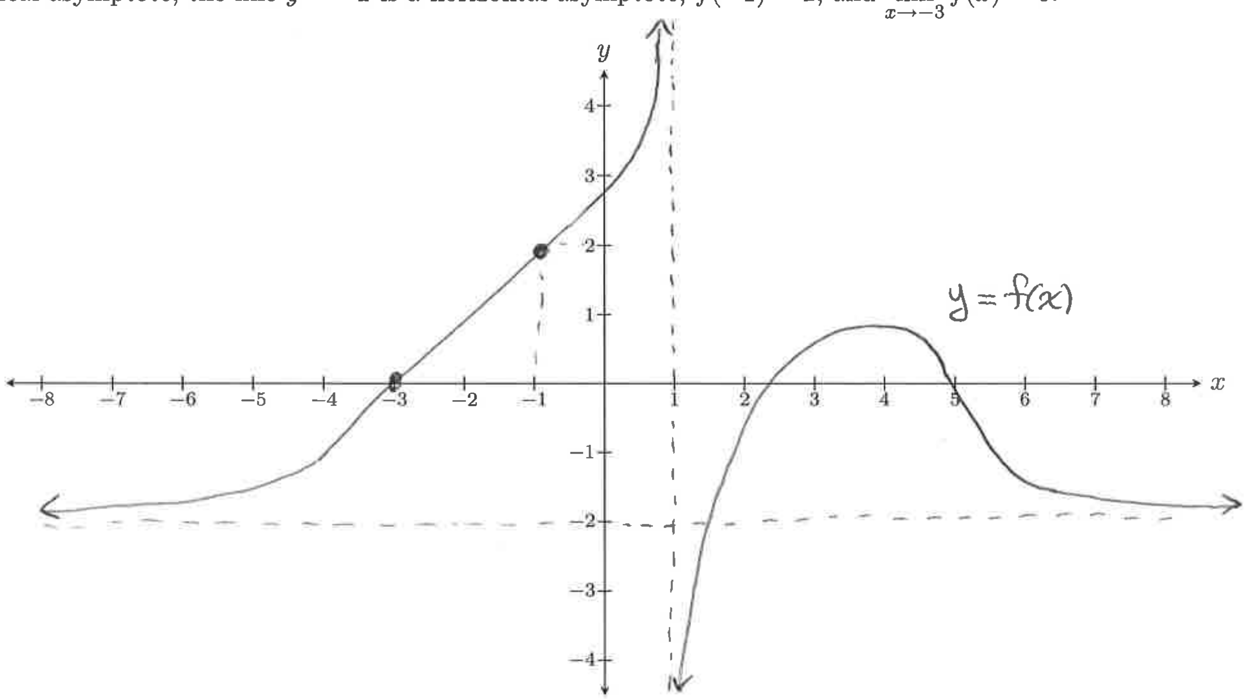
$\dfrac{\sin(x)}{\cos(x)} = \pm\sqrt{3} = \dfrac{\pm\frac{\sqrt{3}}{2}}{\frac{1}{2}}$



By the unit circle, the values of x for which $\frac{\sin(x)}{\cos(x)} = \pm\sqrt{3}$ are

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

3. (10 pts.) Sketch the graph of any function $y = f(x)$ that meets the following four criteria: The line $x = 1$ is a vertical asymptote, the line $y = -2$ is a horizontal asymptote, $f(-1) = 2$, and $\lim_{x \to -3} f(x) = 0$.



4. (20 pts.) Answer the following questions about the function $y = f(x)$ graphed below.

(a) $f(1) = \boxed{1}$

(b) $f \circ f(2) = f(f(2)) = f(2) = \boxed{2}$

(c) $\lim_{x \to 0} f(x) = \boxed{3}$

(d) $\lim_{x \to 1} f(x) = \boxed{\text{DNE}}$

(e) $\lim_{x \to 1^+} f(x) = \boxed{2}$

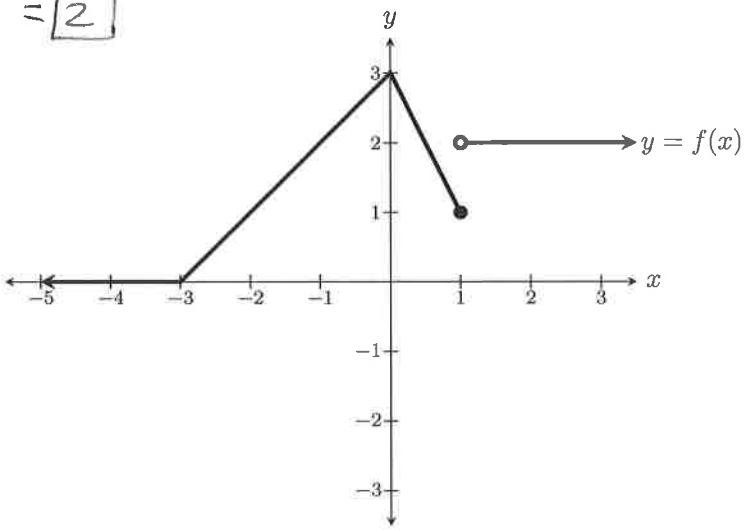
(f) $\lim_{x \to 1^-} f(x) = \boxed{1}$

(g) $\lim_{x \to \infty} f(x) = \boxed{2}$

(h) $\lim_{x \to -\infty} f(x) = \boxed{0}$

(i) State an interval on which $f(x)$ is continuous. $\boxed{(-\infty, 1]}$

(j) State an x-value at which $f(x)$ is discontinuous. $\boxed{1}$



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5. (28 pts.) Evaluate the following limits.
If you want credit, show your steps, explain your reasoning, and carry limits as appropriate.

(a) $\lim_{x \to 2} \frac{x^2 - 7x + 10}{x^2 + x - 6} = \lim_{x \to 2} \frac{(x-2)(x-5)}{(x-2)(x+3)} = \lim_{x \to 2} \frac{x-5}{x+3} = \frac{2-5}{2+3} = \boxed{-\frac{3}{5}}$

(b) $\lim_{h \to 0} \frac{\sqrt{7-h} - \sqrt{7}}{h} = \lim_{h \to 0} \frac{\sqrt{7-h} - \sqrt{7}}{h} \frac{\sqrt{7-h} + \sqrt{7}}{\sqrt{7-h} + \sqrt{7}} = \lim_{h \to 0} \frac{\cancel{7+h} - 7}{h(\sqrt{7-h} + \sqrt{7})}$

$= \lim_{h \to 0} \frac{-h}{h(\sqrt{7-h} + \sqrt{7})} = \lim_{h \to 0} \frac{-1}{\sqrt{7-h} + \sqrt{7}} = \frac{-1}{\sqrt{7-0} + \sqrt{7}} = \boxed{\frac{-1}{2\sqrt{7}}}$

(c) $\lim_{x \to 4^-} \frac{(-x+4)(x+3)}{|-x+4|} = \lim_{x \to 4^-} \frac{-x+4}{|-x+4|}(x+3) = (1)(4+3) = \boxed{7}$

Note: $\frac{-x+4}{|-x+4|} = 1$ when x is to the left of 4 (e.g. $x=3$)

(d) $\lim_{\theta \to 0} \frac{1}{\theta \cot(4\theta)} =$

$= \lim_{\theta \to 0} \frac{1}{\theta \frac{\cos(4\theta)}{\sin(4\theta)}} = \lim_{\theta \to 0} \frac{1}{\frac{\theta}{\sin(4\theta)} \frac{\cos(4\theta)}{1}} = \lim_{\theta \to 0} \frac{\sin(4\theta)}{\theta} \frac{1}{\cos(4\theta)}$

$= \lim_{\theta \to 0} 4 \frac{\sin(4\theta)}{4\theta} \frac{1}{\cos(4\theta)} = 4(1)\frac{1}{\cos(0)} = 4 \cdot 1 \cdot \frac{1}{1} = \boxed{4}$

6. (12 pts.) Find all the horizontal asymptotes and vertical asymptotes of $f(x) = \frac{x^2+x-6}{x^2-x-12}$. $= \frac{(x-2)(x+3)}{(x-4)(x+3)} = \frac{x-2}{x-4}$

To find the horizontal asymptotes, we investigate the limit

$$\lim_{x\to\infty} \frac{x^2+x-6}{x^2-x-12} = \lim_{x\to\infty} \frac{x^2+x-6}{x^2-x-12}\frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{1+\frac{1}{x}-\frac{6}{x^2}}{1-\frac{1}{x}-\frac{12}{x^2}} = \frac{1+0+0}{1+0+0} = \frac{1}{1} = 1.$$

Similarly $\lim_{x\to-\infty} f(x) = 1$. Therefore $\boxed{\text{the line } y=1 \text{ is a H.A.}}$

The candidates for the vertical asymptotes are $x=4$ & $x=-3$, as the numbers 4 and -3 make the denominator zero. Checking:

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \frac{\overbrace{x-2}^{\text{approaches 2}}}{\underbrace{x-4}_{\text{shrinks to zero, positive}}} = \infty$$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \frac{x-2}{x-4} = \frac{-3-2}{-3-4} = \frac{5}{7} \neq \pm\infty$$

$\boxed{\text{Thus } x=4 \text{ is the only V.A}}$

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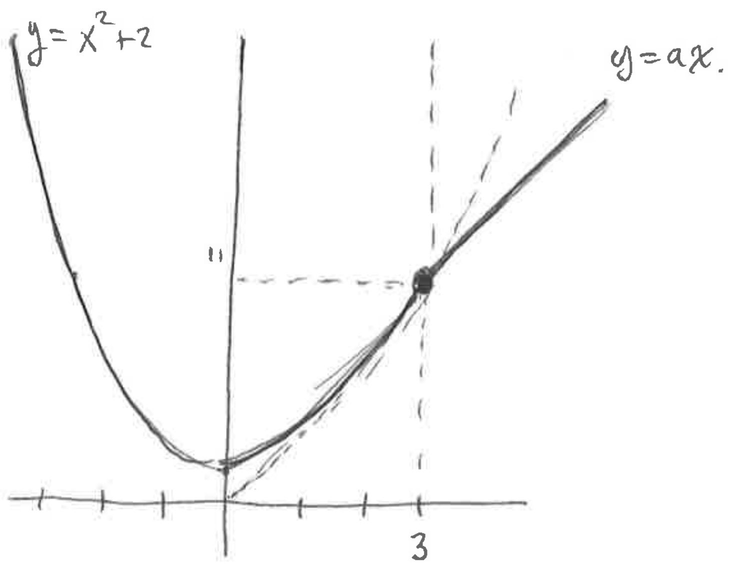
7. (10 pts.) Find the value a such that the following $f(x)$ is continuous at every number x.

$$f(x) = \begin{cases} x^2+2 & \text{if } x < 3 \\ ax & \text{if } x \geq 3 \end{cases}$$



The graph of $f(x)$ is $y = x^2+2$ to the left of 3 and $y = ax$ to the right of 3, as illustrated.

If there is to be no jump at $x=3$, $f(3)$ must match with $y = x^2+2$ at 3. That is, we need $f(3) = 3^2+2 = 11$.

Thus $f(3) = a\cdot 3 = 11$

So $\boxed{a = \frac{11}{3}}$