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I'm in the Thurs11 Thurs12 Thurs1 or Fri10 recitation (Circle one)

Sept. 14, 2012

MATH 200 – TEST 1 ♠

1. (10 pts.) This problem concerns the functions $f(x) = \dfrac{\sqrt{x+2}}{\cos(x)+2}$ and $g(x) = \sqrt{x} - 2$.

   (a) State the domain of $f(x)$. Denominator is positive for any x (never 0) so no problems there. However the $x+2$ in the radical must ~~[illegible]~~ not be negative, that is, we require $x+2 \geq 0$, or $x \geq -2$. Thus $\boxed{\text{The domain is } [-2,\infty)}$

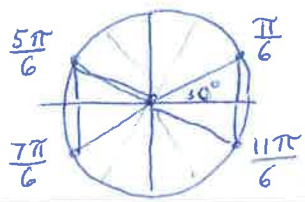
   (b) $f \circ g(x) = f(g(x)) = \dfrac{\sqrt{g(x)+2}}{\cos(g(x))+2} = \dfrac{\sqrt{\sqrt{x}-2+2}}{\cos(\sqrt{x}-2)+2} = \dfrac{\sqrt{\sqrt{x}}}{\cos(\sqrt{x}-2)+2} = \dfrac{\sqrt[4]{x}}{\cos(\sqrt{x}-2)+2}$

2. (10 pts.) Consider the equation $4\sin^2(x) - 1 = 0$. Find all solutions x that lie in the interval $[0, 2\pi)$.

$4\sin^2(x) = 1$

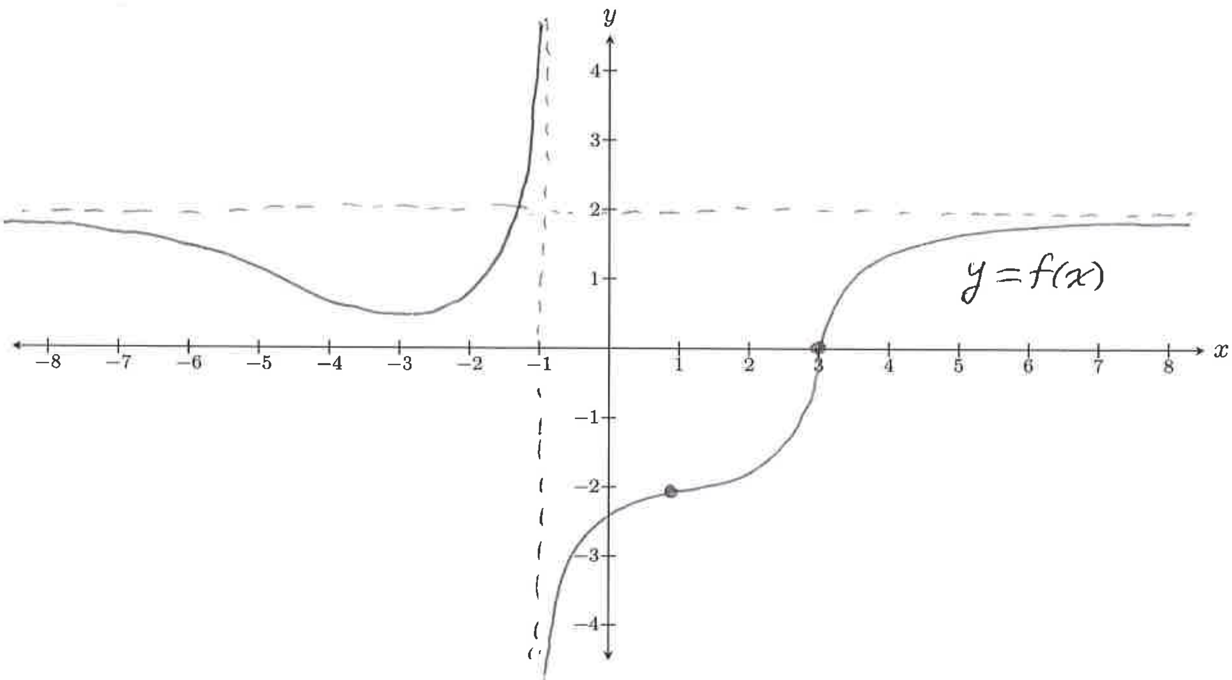
$\sin^2(x) = \frac{1}{4}$

$\sin(x) = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$



Using the unit circle, we see that the four values of x that make $\sin(x) = \pm\frac{1}{2}$ are $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}}$

3. (10 pts.) Sketch the graph of any function $y = f(x)$ that meets the following four criteria: The line $x = -1$ is a vertical asymptote, the line $y = 2$ is a horizontal asymptote, $f(1) = -2$, and $\lim_{x\to 3} f(x) = 0$.



4. (20 pts.) Answer the following questions about the function $y = f(x)$ graphed below.

(a) $f(1) =$ $\boxed{0}$

(b) $f \circ f(2) = f(f(2)) = f(1) = \boxed{0}$

(c) $\lim_{x\to 0} f(x) =$ $\boxed{2}$

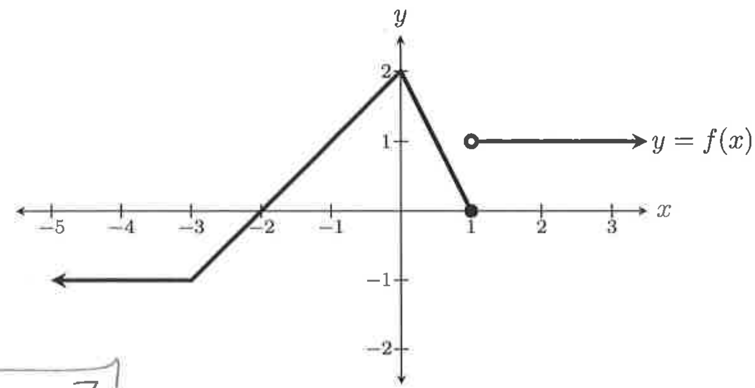
(d) $\lim_{x\to 1} f(x) =$ $\boxed{\text{DNE}}$

(e) $\lim_{x\to 1^+} f(x) =$ $\boxed{1}$

(f) $\lim_{x\to 1^-} f(x) =$ $\boxed{0}$

(g) $\lim_{x\to \infty} f(x) =$ $\boxed{1}$

(h) $\lim_{x\to -\infty} f(x) =$ $\boxed{-1}$



(i) State an interval on which $f(x)$ is continuous. $\boxed{(-\infty, 1]}$

(j) State an x-value at which $f(x)$ is discontinuous. $\boxed{x = 1.}$

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5. (28 pts.) Evaluate the following limits.
If you want credit, show your steps, explain your reasoning, and carry limits as appropriate.

(a) $\lim_{x\to 1} \dfrac{x^2 - 4x + 3}{x^2 + 4x - 5} = \lim_{x\to 1} \dfrac{(x-1)(x-3)}{(x-1)(x+5)} = \lim_{x\to 1} \dfrac{x-3}{x+5} = \dfrac{1-3}{1+5} = \dfrac{-2}{6} = \boxed{-\dfrac{1}{3}}$

(b) $\lim_{h\to 0} \dfrac{\sqrt{5+h} - \sqrt{5}}{h} = \lim_{h\to 0} \dfrac{\sqrt{5+h} - \sqrt{5}}{h} \cdot \dfrac{\sqrt{5+h} + \sqrt{5}}{\sqrt{5+h} + \sqrt{5}} = \lim_{h\to 0} \dfrac{5+h-5}{h(\sqrt{5+h} + \sqrt{5})}$

$= \lim_{h\to 0} \dfrac{h}{h(\sqrt{5+h} + \sqrt{5})} = \lim_{h\to 0} \dfrac{1}{\sqrt{5+h} + \sqrt{5}} = \dfrac{1}{\sqrt{5+0} + \sqrt{5}} = \boxed{\dfrac{1}{2\sqrt{5}}}$

(c) $\lim_{x\to 4^+} \dfrac{(-x+4)(x+2)}{|-x+4|} = \lim_{x\to 4^+} \dfrac{-x+4}{|-x+4|}(x+2) = \lim_{x\to 4^+} (-1)(x+2) = -(4+2) = \boxed{-6}$

$\dfrac{-x+4}{|-x+4|} = -1$ when x is to the right of 4

(d) $\lim_{\theta\to 0} \dfrac{1}{\theta} \tan(3\theta) = \lim_{\theta\to 0} \dfrac{1}{\theta} \dfrac{\sin(3\theta)}{\cos(3\theta)} = \lim_{\theta\to 0} \dfrac{\sin(3\theta)}{\theta} \dfrac{1}{\cos(3\theta)}$

$= \lim_{\theta\to 0} 3 \dfrac{\sin(3\theta)}{3\theta} \dfrac{1}{\cos(3\theta)} = 3 \cdot 1 \cdot \dfrac{1}{\cos(0)} = 3 \cdot 1 \cdot \dfrac{1}{1} = \boxed{3}$

6. (12 pts.) Find all the horizontal asymptotes and vertical asymptotes of $f(x) = \dfrac{x^2+x-2}{x^2-x-6} = \dfrac{(x-1)(x+2)}{(x-3)(x+2)} = \dfrac{x-1}{x-3}$

To find the horizontal asymptotes, we investigate the following limit:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2+x-2}{x^2-x-6}\,\frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{1+1/x-2/x^2}{1-1/x-6/x^2} = \frac{1+0+0}{1+0+0} = \frac{1}{1} = 1.$$

Similarly, $\lim_{x\to-\infty} f(x) = 1.$ $\boxed{\text{Therefore } y=1 \text{ (i.e. the line } y=1\text{) is the H.A.}}$

The candidates for the vertical asymptotes are $x=3$ and $x=-2$ (by above factoring.) Checking these,

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{\overset{\text{near } 2}{x-1}}{\underset{\text{near } 0, \text{ pos.}}{x-3}} = \infty$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{x-1}{x-3} = \frac{-2-1}{-2-3} = \frac{3}{5} \neq \infty$$

From this, we see that $\boxed{\text{the line } x=3 \text{ is the only vertical asymptote.}}$

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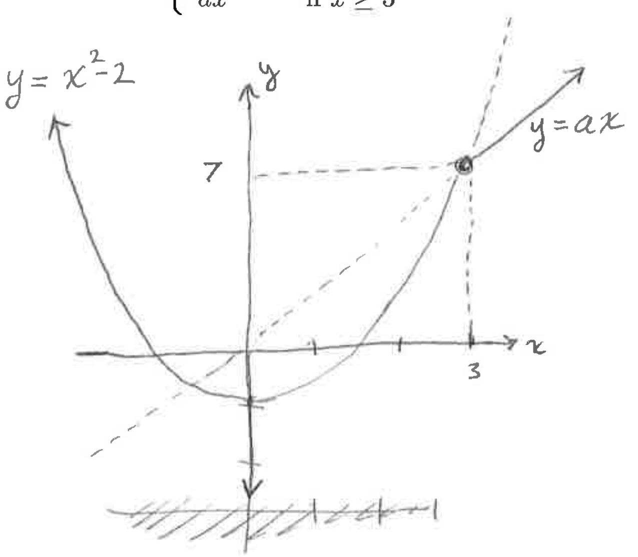
7. (10 pts.) Find the value a such that the following $f(x)$ is continuous at every number x.

$$f(x) = \begin{cases} x^2-2 & \text{if } x < 3 \\ ax & \text{if } x \geq 3 \end{cases}$$



If f is to be continuous, its graph must look as on the left: It is the parabola x^2-2 to the left of 3 and the line $y=ax$ to the right of 3, AND the point $(3,7)$ on $y=x^2-7$ must be on the graph of $y=ax$ (so there are no jumps).

Thus we must have

$$y = ax$$

$$7 = a\cdot 3$$

$$\boxed{a = \frac{7}{3}}.$$