MATH 200
CALCULUS I

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TEST 1

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Score: 100

Directions. Please solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not to be used.

6. (15 points) Answer the questions about the function $f(x)$ graphed below.

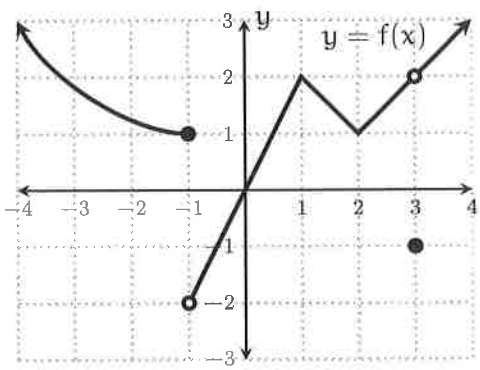
(a) $\lim_{x\to -1^+} f(x) = \boxed{-2}$

(b) $\lim_{x\to -1^-} f(x) = \boxed{1}$

(c) $\lim_{x\to 3} \dfrac{5f(x)}{1+f(x)} = \dfrac{\lim_{x\to 3} 5f(x)}{\lim_{x\to 3}(1+f(x))} = \dfrac{5\cdot 2}{1+2} = \boxed{\dfrac{10}{3}}$

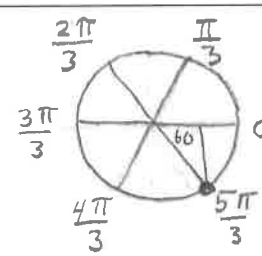
(d) $f\circ f(1) = f(f(1)) = f(2) = \boxed{1}$

(e) At which values c is $f(x)$ **not** continuous at $x = c$? $\boxed{-1 \text{ and } 3}$

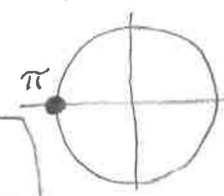


1. (25 points) Warmup: short answer.

(a) $\tan(5\pi/3) = \dfrac{\sin(5\pi/3)}{\cos(5\pi/3)} = \dfrac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{-\sqrt{3}}$



(c) Describe the domain of $f(x) = \dfrac{x}{1+\cos(x)}$. Only problem would be $\cos(x) = -1$, which makes the denominator zero. For this, $x = \pi + 2k\pi$. Domain: $\boxed{\text{All real numbers except } \pi + 2k\pi \text{ for } k = 0, \pm 1, \pm 2, \ldots}$

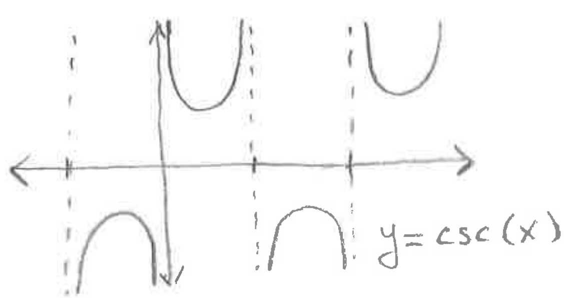


(e) If $f(x) = \sec(x)\tan(x)$ and $g(x) = \dfrac{x}{\cos(x)}$,

then $f\circ g(x) = \boxed{\sec\left(\dfrac{x}{\cos(x)}\right)\tan\left(\dfrac{x}{\cos(x)}\right)}$

(b) $\lim_{x\to 27}\left(1 + x^{2/3}\right) = 1 + 27^{2/3} = 1 + \sqrt[3]{27}^{\,2} = 1 + 3^2 = \boxed{10}$

(e) $\lim_{x\to 0^-} \csc(x) = \lim_{x\to 0^-} \dfrac{1}{\sin(x)} = \boxed{-\infty}$



2. (15 points) Find all solutions of the equation

$2x\sin(x) + x = 0$, where $-\pi \leqslant x \leqslant \pi$.

$x(2\sin(x)+1) = 0$ (factor out x)

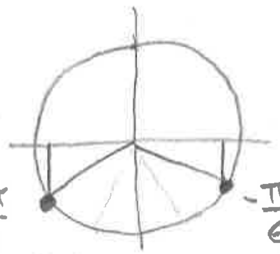
$x = 0$

$2\sin(x) + 1 = 0$

$2\sin(x) = -1$

$\sin(x) = -\frac{1}{2}$

$x = -\frac{\pi}{6}, -\frac{5\pi}{6}$



Answer: $x = 0, -\frac{\pi}{6}, -\frac{5\pi}{6}$

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3. (15 points) Sketch the graph of any function that meets the following criteria.

(a) $f(3) = 2$

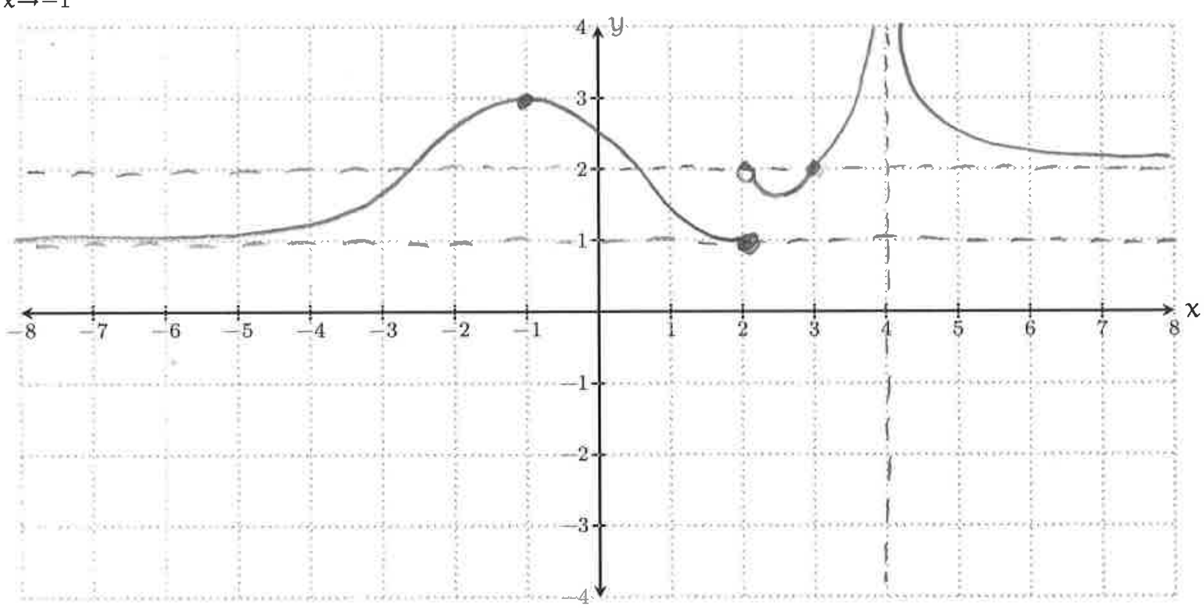
(b) Lines $y = 2$ and $y = 1$ are horizontal asymptotes.

(c) $\lim_{x \to 4} f(x) = \infty$ (so the line $x = 4$ is a vertical asymptote)

(d) $\lim_{x \to 1^+} f(x) = 2$

(e) $\lim_{x \to 1^-} f(x) = 1$

(f) $\lim_{x \to -1} f(x) = 3$



Here is one of many correct solutions.

4. (15 points) Evaluate the following limits.

(a) $\lim_{x\to 5} \frac{x^2-3x-10}{x^2-8x+15} = \lim_{x\to 5} \frac{(x+2)\cancel{(x-5)}}{(x-3)\cancel{(x-5)}} = \lim_{x\to 5} \frac{x+2}{x-3} = \frac{5+2}{5-3} = \boxed{\frac{7}{2}}$

(b) $\lim_{x\to 0} \frac{(x-3)\sin(x)}{2x^2-6x} = \lim_{x\to 0} \frac{(x-3)\sin(x)}{(2x-6)\,x} = \lim_{x\to 0} \frac{(x-3)\sin(x)}{2(x-3)\,x}$

$= \lim_{x\to 0} \frac{1}{2}\,\frac{\sin(x)}{x} = \frac{1}{2}\cdot 1 = \boxed{\frac{1}{2}}$

(c) $\lim_{h\to 0} \frac{\sqrt{6+h}-\sqrt{6}}{h} = \lim_{h\to 0} \frac{\sqrt{6+h}-\sqrt{6}}{h}\;\frac{\sqrt{6+h}+\sqrt{6}}{\sqrt{6+h}+\sqrt{6}}$

$= \lim_{h\to 0} \frac{6+h-6}{h(\sqrt{6+h}+\sqrt{6})} = \lim_{h\to 0} \frac{h}{h(\sqrt{6+h}+\sqrt{6})} = \lim_{h\to 0} \frac{1}{\sqrt{6+h}+\sqrt{6}} = \boxed{\frac{1}{2\sqrt{6}}}$

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5. (15 points) This question concerns the function

$f(x) = \frac{x^2-4}{5x^2-10x}. = \frac{(x+2)(x-2)}{5x(x-2)} = \frac{x+2}{5x}$ If $x \neq 2$

(a) State the intervals on which $f(x)$ is continuous.

Rational function is continuous on its domain: $\boxed{(-\infty, 0) \cup (0, 2) \cup (2, \infty)}$

(b) Find the horizontal asymptotes (if any).

$\lim_{x\to\infty} f(x) = \frac{1}{5}$, so $\boxed{\text{the line } y = \frac{1}{5} \text{ is a H. A}}$

(c) Find the vertical asymptotes (if any). Candidates are $x=0$ and $x=2$

- $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x+2}{5x} = \infty$ $\boxed{\text{line } x=0 \text{ is V. A}}$
- $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x+2}{5x} = \frac{2+2}{5\cdot 2} = \frac{2}{5} \neq \pm\infty$ (No V. A. here)