MATH 200

CALCULUS I

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TEST 1

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Name: Richard

Score: 100

Directions. Please solve the following questions in the space provided. Unless noted otherwise, you must show your work to receive full credit. This is a closed-book, closed-notes test. Calculators, computers, etc., are not to be used.

6. (15 points) Answer the questions about the function $f(x)$ graphed below.

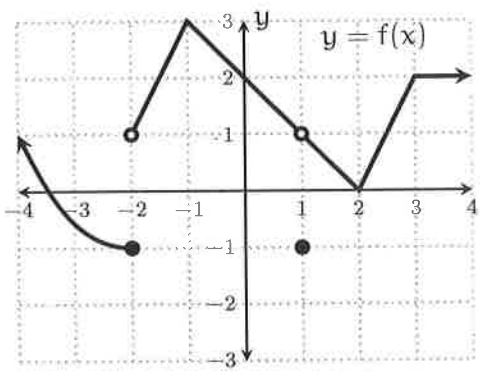
(a) $\lim\limits_{x \to -2^+} f(x) = \boxed{1}$

(b) $\lim\limits_{x \to -2^-} f(x) = \boxed{-1}$

(c) $\lim\limits_{x \to 1} \dfrac{5\,f(x)}{1+f(x)} = \dfrac{5 \lim_{x\to1} f(x)}{1+\lim_{x\to1} f(x)} = \dfrac{5\cdot1}{1+1} = \boxed{\dfrac{5}{2}}$

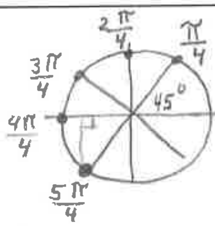
(d) $f \circ f(-1) = f(f(-1)) = f(3) = \boxed{2}$

(e) At which values c is $f(x)$ **not** continuous at $x = c$? graph has hole or jump at $\boxed{x=-2}$ and $\boxed{x=1}$



1. (25 points) Warmup: short answer.

(a) $\sec(5\pi/4) = \dfrac{1}{\cos(5\pi/4)} = \dfrac{1}{-\frac{\sqrt{2}}{2}} = \dfrac{-2}{\sqrt{2}} = \dfrac{-2\sqrt{2}}{\sqrt{2}\sqrt{2}} = \boxed{-\sqrt{2}}$



(b) Describe the domain of $f(x) = \dfrac{x}{1-\tan(x)}$. Only thing that could go wrong is $\tan(x)$ undefined ($x = \frac{\pi}{2} + k\pi$), or $\tan(x) = 1$ ($x = \frac{\pi}{4} + k\pi$), which makes the denominator zero.

Domain: All real numbers except $x = \frac{\pi}{2} + k\pi$ and $x = \frac{\pi}{4} + k\pi$ for $k = 0, \pm1, \pm2, \pm3 \ldots$

(d) If $f(x) = \dfrac{\sin(x)}{x}$ and $g(x) = x + \sqrt{x}$,

then $f \circ g(x) = f(g(x)) = \boxed{\dfrac{\sin(x+\sqrt{x})}{x+\sqrt{x}}}$ NOTE This does NOT equal 1

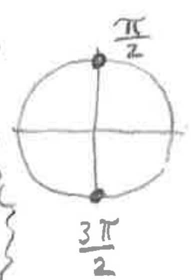
(d) $\lim\limits_{x \to 2} \left(\dfrac{1}{4} + \dfrac{8}{x^2}\right)^{\frac{3}{2}} = \left(\dfrac{1}{4} + \dfrac{8}{2^2}\right)^{3/2} = \sqrt{\dfrac{1}{4} + \dfrac{8}{4}}^{\,3} = \sqrt{\dfrac{9}{4}}^{\,3} = \left(\dfrac{3}{2}\right)^3 = \boxed{\dfrac{27}{8}}$

(e) $\lim\limits_{x \to \frac{\pi}{2}} \cot(x) = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos(x)}{\sin(x)} = \dfrac{\cos(\pi/2)}{\sin(\pi/2)} = \dfrac{0}{1} = \boxed{0}$

2. (15 points) Find all solutions of the equation
$\cos^2(x) - \cos(x)\sin(x) = 0$, where $0 \leqslant x \leqslant 2\pi$.

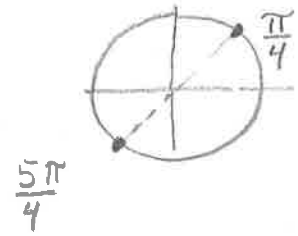
$\cos(x)\left(\cos(x) - \sin(x)\right) = 0$ (factor out $\cos(x)$)

$\cos(x) = 0$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos(x) - \sin(x) = 0$
$\cos(x) = \sin(x)$



$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\boxed{\text{Answer: Solutions are } \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{4}, \frac{\pi}{4}}$

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3. (15 points) Sketch the graph of any function that meets the following criteria.

(a) $f(1) = 2$

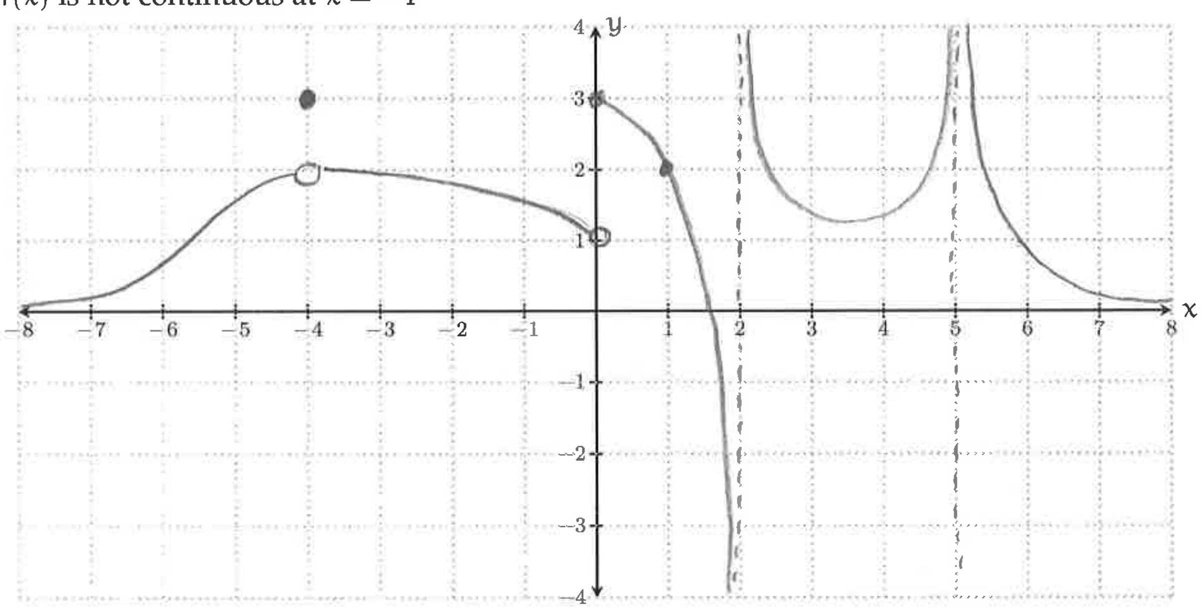
(b) $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$ (So the line $y=0$ is a horizontal asymptote.)

(c) $\lim_{x\to0^+} f(x) = 3$ and $\lim_{x\to0^-} f(x) = 1$

(d) Lines $x = 2$ and $x = 5$ are vertical asymptotes.

(e) $\lim_{x\to-4} f(x) = 2$

(f) $f(x)$ is not continuous at $x = -4$



Here is one of many correct solutions.

4. (15 points) Evaluate the following limits.

(a) $\lim_{x\to 0} \frac{\sin(7x)}{5x} = \lim_{x\to 0} \frac{7}{5} \frac{\sin(7x)}{7x} = \frac{7}{5}\cdot 1 = \boxed{\frac{7}{5}}$

$\frac{0}{0}$

Using $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$

(b) $\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3} = \lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{\sqrt{x}^2-\sqrt{3}^2} = \lim_{x\to 3} \frac{\cancel{\sqrt{x}-\sqrt{3}}}{(\cancel{\sqrt{x}-\sqrt{3}})(\sqrt{x}+\sqrt{3})}$

$\frac{0}{0}$ (Try to cancel!)

$= \lim_{x\to 3} \frac{1}{\sqrt{x}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}}$

(c) $\lim_{h\to 0} \frac{\frac{1}{6+h}-\frac{1}{6}}{h} = \lim_{h\to 0} \frac{\frac{1}{6+h}-\frac{1}{6}}{h} \frac{6(6+h)}{6(6+h)} = \lim_{h\to 0} \frac{6-(6+h)}{h\,6(6+h)}$

$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\,6(6+h)} = \lim_{h\to 0} \frac{-1}{6(6+h)} = \frac{-1}{6(6+0)} = \boxed{\frac{-1}{36}}$

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5. (15 points) This question concerns the function

$f(x) = \frac{x^2-1}{7x^3-7x^2}$. $= \frac{(x-1)(x+1)}{7x^2(x-1)} = \frac{x+1}{7x^2}$

Provided $x \neq 1$

(a) State the intervals on which $f(x)$ is continuous.

Rational function $f(x)$ is $\boxed{\text{continuous on its domain } (-\infty, 0) \cup (0,1) \cup (1,\infty)}$

(b) Find the horizontal asymptotes (if any).

$\lim_{x\to\infty} \frac{x^2-1}{7x^3-7x^2} = 0$ so $\boxed{\text{line } y=0 \text{ is a H.A.}}$

(c) Find the vertical asymptotes (if any). Candidates: $x=0$ and $x=1$.

Check $x=0$: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{x+1}{7x^2} = \infty$. $\boxed{\text{Line } x=0 \text{ is V.A.}}$

Check $x=1$: $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{1+1}{7\cdot 1^2} = \frac{2}{7} \neq \infty$ (No V.A. here!)